Statistics for Astronomers
Homework #5 (Due before 5:00 PM on Monday, 2019.04.29)

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1. **(5 points)**
   You toss a coin N times, obtaining k heads. Unsure about whether the coin is fair, you assume that the probability of obtaining a head, θ, is Beta(α, β), with $\alpha, \beta > 0$.

   (a) What is the prior mean of θ?

   (b) What is the posterior probability distribution of θ?

   (c) What is the posterior mean of θ?

   (d) What is the effective sample size?

2. **(5 points)**
   Suppose we draw N random deviates X_i $(i = 1, \cdots, N)$ from a normal distribution with known population standard deviation σ.

   (a) Derive the Jeffreys prior for μ.

   (b) Use the prior you just computed to find the posterior probability distribution for μ in terms of the data values x_i, N, and σ.

3. **(7 points)**

   (a) Develop a script to compute the highest posterior density (HPD) interval using the algorithm discussed in Lecture 14.

   (b) Use this script to compute the HPD interval for the source counts example discussed during the same lecture.

4. **(10 points)**
   $N = 4$ random deviates X_i $(i = 1, \cdots, N)$ are drawn from a Poisson distribution with unknown rate parameter λ, resulting in a sample mean of $\bar{X} = 0.5$.

   (a) Compute the Jeffreys prior for λ. Is it an improper prior?

   (b) Compute the posterior probability distribution for λ, normalise it, and find the mode, mean, and variance.

   (c) Compute the HPD interval for the posterior using the script from part (3a).

   (d) Discuss how the results to parts (4b) and (4c) would change if $N = 40$.