Statistics for Astronomers
Homework #6 (Due before 5:00 PM on Friday, 2019.05.17)

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1. **(4 points)** Given $N = 101$ points, evaluate $J(N)$ and $J(2N)$, the Simpson's Rule estimates for the integral

$$J = \int_{-1}^{+1} dx\, \sqrt{1-x^2}\, e^{-x}.$$

2. **(6 points)**
The flux density $F_{\nu,\mathrm{BB}}(\lambda)$ in mJy at wavelength $\lambda = 7.872$ μm received from a blackbody is given by

$$F_{\nu,\mathrm{BB}}(\lambda) = \frac{0.26184\left(\dfrac{R_{\mathrm{eff}}}{D}\right)^2}{\exp\left(\dfrac{1848.9}{T_{\mathrm{eff}}}\right) - 1},$$

with R_{eff} in AU, D in kpc, and T_{eff} in K. Assume that the values for (R_{eff}, D, and T_{eff}) are normally distributed about means (1.330 AU, 0.892 kpc, 3393 K) with standard deviations equal to (0.300 AU, 0.092 kpc, 275 K).

   (a) Find the mean, median, and mode for the distribution of fluxes generated by propagating the uncertainties in the radius, distance, and temperature.

   (b) What are the upper and lower limits for the 68% equal-tailed interval?

3. **(15 points)**
Read through the procedure for rejection sampling described here.
In this problem, you will use rejection sampling to draw random variables from the blackbody distribution (this will be our **target** distribution)

$$p(x) \propto \frac{x^3}{e^x - 1}.$$

The **proposal distribution** for this problem will be a normal distribution.

   (a) Choose appropriate values of the mean and standard deviation for the proposal distribution.

   (b) Choose an appropriate scaling factor to multiply into the proposal distribution so it is everywhere $\geq$ the target.

(c) Starting with $N = 10\,000$ draws from the proposal distribution, write a script that uses rejection sampling to simulate draws from the target distribution.

(d) As part of the same script, generate a kernel density estimate (you can use `statsmodels.nonparametric.kde.KDEUnivariate`) from the simulated draws. Compare this to the target distribution function on a plot to verify that it has the proper shape.