Statistics for Astronomers
Homework #7 (Due before 12:00 PM on Monday, 2021.01.18)

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Notes: (1) You are welcome to use `Python` functions to evaluate probabilities for various distributions, and `Mathematica/Wolfram Alpha` to compute integrals if necessary. **Just mention your source in each case!** (2) Email me your `Python` scripts and any/all resulting output plots/images.

1. (**6 points**) Derive the Jeffreys priors for the population mean and population standard deviation of a normally-distributed random variable.

2. Tossing a coin $N = 1000$ times results in $x = 473$ heads. Let θ be the probability of obtaining a head from a single coin toss. Consider two scenarios: (a) θ has a Uniform prior (b) θ has the Jeffreys prior. Answer the following questions using the properties of the Beta distribution and the fact that

$$\int_0^1 d\theta\ \theta^x\ (1-\theta)^{N-x} = \frac{\Gamma(x+1)\ \Gamma(N-x+1)}{\Gamma(N+2)}.$$

   (a) (**2 points**) Derive the Jeffreys prior for θ.

   (b) (**3 points**) For each prior, what are the mean and variance? Based on these values, explain which prior is less informative and why.

   (c) (**4 points**) What is the prior predictive distribution for x for each prior?

   (d) (**7 points**) What is the posterior distribution of θ for each prior? For each prior, what are the posterior mean and variance? Is the difference in the distributions and the means/variances significant? What is the reason for this?

   (e) (**4 points**) For each prior, what is the posterior probability that $\theta > 0.5$?

   (f) (**4 points**) For each prior, what is the probability that, following the above experiment, a single coin toss results in a head?