



Stellar Atmospheres: Lecture 2, 2020.04.15

Prof. Sundar Srinivasan

IRyA/UNAM



Review: bolometric magnitude and bolometric correction

Bolometer – measures incident **power** (regardless of frequency of incident photons) by heating a material whose electrical resistance is temperature-dependent.

Review: bolometric magnitude and bolometric correction

Bolometer – measures incident **power** (regardless of frequency of incident photons) by heating a material whose electrical resistance is temperature-dependent.

Total incident power = Area of detector \times incident integrated flux $\propto \int_0^{\infty} d\nu F_{\nu}$.

The integrated flux is therefore also called the **bolometric flux**, and is represented by F_{bol} .

Review: bolometric magnitude and bolometric correction

Bolometer – measures incident **power** (regardless of frequency of incident photons) by heating a material whose electrical resistance is temperature-dependent.

Total incident power = Area of detector \times incident integrated flux $\propto \int_0^{\infty} d\nu F_{\nu}$.

The integrated flux is therefore also called the **bolometric flux**, and is represented by F_{bol} .

Bolometric magnitudes can then be defined similar to magnitudes in a broadband filter:

$m_{\text{bol}} = -2.5 \log F_{\text{bol}}$ (app.), and $M_{\text{bol}} = -2.5 \log L_{\text{bol}}$ (abs.), so that $\mu = m_{\text{bol}} - M_{\text{bol}}$ holds.

Review: bolometric magnitude and bolometric correction

Bolometer – measures incident **power** (regardless of frequency of incident photons) by heating a material whose electrical resistance is temperature-dependent.

Total incident power = Area of detector \times incident integrated flux $\propto \int_0^{\infty} d\nu F_{\nu}$.

The integrated flux is therefore also called the **bolometric flux**, and is represented by F_{bol} .

Bolometric magnitudes can then be defined similar to magnitudes in a broadband filter:
 $m_{\text{bol}} = -2.5 \log F_{\text{bol}}$ (app.), and $M_{\text{bol}} = -2.5 \log L_{\text{bol}}$ (abs.), so that $\mu = m_{\text{bol}} - M_{\text{bol}}$ holds.

A broadband filter has a finite bandwidth (range of accepted wavelengths), so it only detects a fraction of F_{bol} .

For a filter A , if we assume that $F_A \times \Delta\lambda = \text{const.} \times F_{\text{bol}}$ or $F_{\text{bol}} = \text{const.} \times F_A$, then (taking the distance to the source into account), we write the absolute bolometric magnitude as

$$M_{\text{bol}} = M_A + BC.$$

Review: bolometric magnitude and bolometric correction

Bolometer – measures incident **power** (regardless of frequency of incident photons) by heating a material whose electrical resistance is temperature-dependent.

Total incident power = Area of detector \times incident integrated flux $\propto \int_0^{\infty} d\nu F_{\nu}$.

The integrated flux is therefore also called the **bolometric flux**, and is represented by F_{bol} .

Bolometric magnitudes can then be defined similar to magnitudes in a broadband filter:
 $m_{\text{bol}} = -2.5 \log F_{\text{bol}}$ (app.), and $M_{\text{bol}} = -2.5 \log L_{\text{bol}}$ (abs.), so that $\mu = m_{\text{bol}} - M_{\text{bol}}$ holds.

A broadband filter has a finite bandwidth (range of accepted wavelengths), so it only detects a fraction of F_{bol} .

For a filter A , if we assume that $F_A \times \Delta\lambda = \text{const.} \times F_{\text{bol}}$ or $F_{\text{bol}} = \text{const.} \times F_A$, then (taking the distance to the source into account), we write the absolute bolometric magnitude as

$M_{\text{bol}} = M_A + BC$. The quantity BC is called the **bolometric correction**.

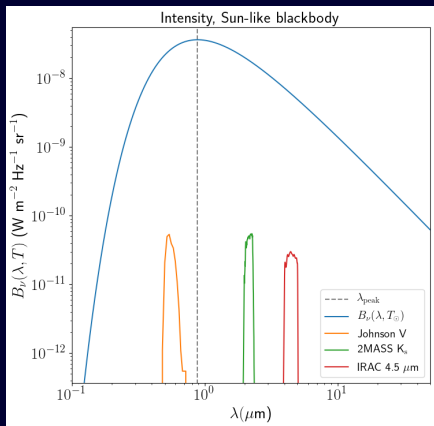
In practice, we usually have one broadband magnitude. If we know the BC in that band for stars of a certain type, we can estimate their luminosities.

The absolute magnitude in one band \rightarrow observational proxy for the luminosity.

Review: bolometric magnitude and bolometric correction

Ideal: filter choice minimises BC for target star.

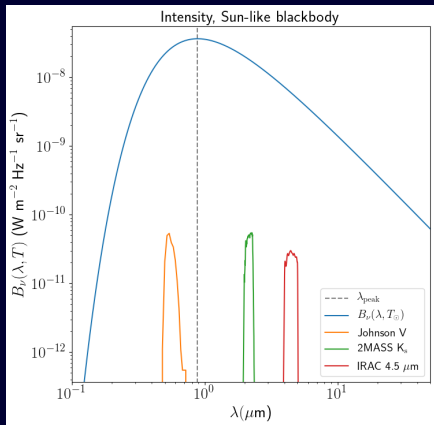
Review: bolometric magnitude and bolometric correction



BC as a function of wavelength

Ideal: filter choice minimises *BC* for target star.

Review: bolometric magnitude and bolometric correction



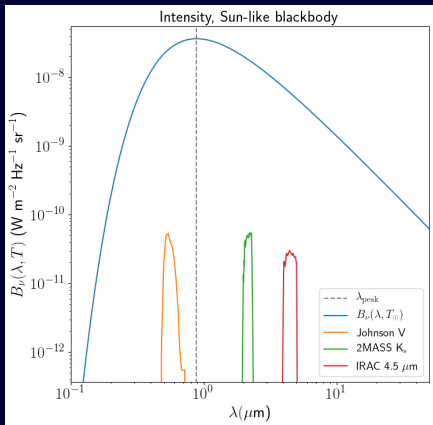
BC as a function of wavelength

Ideal: filter choice minimises BC for target star.

<http://mips.as.arizona.edu/~cnaw/sun.html>

Compare $M_{\text{bol}, \odot} = 4.74$ mag to the value in the following filters:
Johnson V, 2MASS K_s , Spitzer IRAC 4.5 μm .

Review: bolometric magnitude and bolometric correction



BC as a function of wavelength

Ideal: filter choice minimises BC for target star.

<http://mips.as.arizona.edu/~cnaw/sun.html>

Compare $M_{\text{bol}, \odot} = 4.74$ mag to the value in the following filters:
Johnson V , 2MASS K_s , *Spitzer* IRAC 4.5 μm .

Of the three, the V -band is closest to λ_{peak} .

Results will change if location of λ_{peak} changes w.r.t. filters (Homework #1).

Review: Effective temperature and colour

$T(\text{star})$ changes with distance from the centre, but we only see radiation from its surface and beyond.

Assume blackbody, use Stefan-Boltzmann Law to define an **effective temperature**:

$$L = 4\pi\sigma R^2 T_{\text{eff}}^4, \text{ where } R \text{ is the radius of the photosphere.}$$

T_{eff} is then also the temperature at the surface of the star.

Review: Effective temperature and colour

T (star) changes with distance from the centre, but we only see radiation from its surface and beyond.

Assume blackbody, use Stefan-Boltzmann Law to define an **effective temperature**:

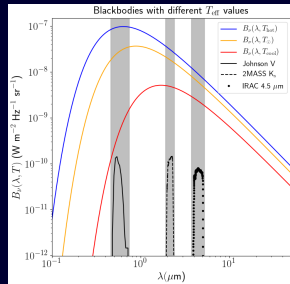
$$L = 4\pi\sigma R^2 T_{\text{eff}}^4, \text{ where } R \text{ is the radius of the photosphere.}$$

T_{eff} is then also the temperature at the surface of the star.

From Wien's Displacement Law, $\lambda_{\text{peak}} \times T_{\text{eff}} = \text{constant}$

\Rightarrow **cooler stars peak at longer wavelengths.**

\Rightarrow we can use the terms "bluer" and "redder" when comparing T_{eff} .



Broadband fluxes as a function of T_{eff}

Review: Effective temperature and colour

$T(\text{star})$ changes with distance from the centre, but we only see radiation from its surface and beyond.

Assume blackbody, use Stefan-Boltzmann Law to define an **effective temperature**:

$$L = 4\pi\sigma R^2 T_{\text{eff}}^4, \text{ where } R \text{ is the radius of the photosphere.}$$

T_{eff} is then also the temperature at the surface of the star.

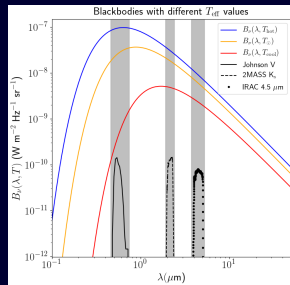
From Wien's Displacement Law, $\lambda_{\text{peak}} \times T_{\text{eff}} = \text{constant}$

\Rightarrow **cooler stars peak at longer wavelengths.**

\Rightarrow we can use the terms "bluer" and "redder" when comparing T_{eff} .

Colour = ratio of fluxes at two wavelengths.

Ratio in linear space \rightarrow difference in log-space. So,



Broadband fluxes as a function of T_{eff}

Review: Effective temperature and colour

$T(\text{star})$ changes with distance from the centre, but we only see radiation from its surface and beyond.

Assume blackbody, use Stefan-Boltzmann Law to define an **effective temperature**:

$$L = 4\pi\sigma R^2 T_{\text{eff}}^4, \text{ where } R \text{ is the radius of the photosphere.}$$

T_{eff} is then also the temperature at the surface of the star.

From Wien's Displacement Law, $\lambda_{\text{peak}} \times T_{\text{eff}} = \text{constant}$

\Rightarrow **cooler stars peak at longer wavelengths.**

\Rightarrow we can use the terms "bluer" and "redder" when comparing T_{eff} .

Colour = ratio of fluxes at two wavelengths.

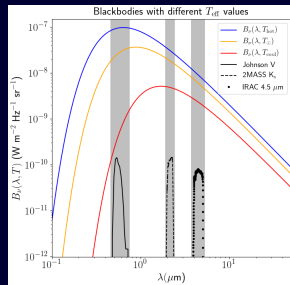
Ratio in linear space \rightarrow difference in log-space. So,

$$T_{\text{eff,hot}} > T_{\text{eff},\odot} > T_{\text{eff,cool}}$$

$$\Rightarrow \left(\frac{F_V}{F_{4.5}}\right)_{\text{hot}} > \left(\frac{F_V}{F_{4.5}}\right)_{\odot} > \left(\frac{F_V}{F_{4.5}}\right)_{\text{cool}}$$

$$\Rightarrow (m_V - m_{4.5})_{\text{hot}} < (m_V - m_{4.5})_{\odot} < (m_V - m_{4.5})_{\text{cool}}$$

(note reversal!)



Broadband fluxes as a function of T_{eff}

Connecting observations to theory

The characteristics of stellar spectra (absolute flux, continuum shape, emission/absorption features) depend on the physical parameters in the stellar atmosphere as well as the relationship between these parameters.

Connecting observations to theory

The characteristics of stellar spectra (absolute flux, continuum shape, emission/absorption features) depend on the physical parameters in the stellar atmosphere as well as the relationship between these parameters.

These parameters influence the chemical composition, distribution of velocities, and excitation/ionisation states of various species in the atmosphere.

Connecting observations to theory

The characteristics of stellar spectra (absolute flux, continuum shape, emission/absorption features) depend on the physical parameters in the stellar atmosphere as well as the relationship between these parameters.

These parameters influence the chemical composition, distribution of velocities, and excitation/ionisation states of various species in the atmosphere.

We will now focus on general results and observational proxies for these parameters.

Basic assumptions

Unless otherwise specified, we will assume the following:

- 1 **Ideal gas:** reasonable assumption for most regions of the star.

Basic assumptions

Unless otherwise specified, we will assume the following:

- 1 **Ideal gas:** reasonable assumption for most regions of the star.

$pV_{\text{mol}} = \mathcal{R}T$, where V_{mol} is the molar volume (volume occupied by 1 mole of gas).

Basic assumptions

Unless otherwise specified, we will assume the following:

- 1 **Ideal gas:** reasonable assumption for most regions of the star.

$pV_{\text{mol}} = \mathcal{R}T$, where V_{mol} is the molar volume (volume occupied by 1 mole of gas).

$\Rightarrow p = \rho \frac{kT}{\mu m_p}$, with μm_p the **mean molecular weight** (mass of a “typical” particle in m_p).

Basic assumptions

Unless otherwise specified, we will assume the following:

- 1 **Ideal gas:** reasonable assumption for most regions of the star.

$pV_{\text{mol}} = \mathcal{R}T$, where V_{mol} is the molar volume (volume occupied by 1 mole of gas).

$\Rightarrow p = \rho \frac{kT}{\mu m_p}$, with μm_p the **mean molecular weight** (mass of a “typical” particle in m_p).

- 2 **Hydrostatic equilibrium:** $\nabla p = -\rho g$, where $g = \frac{GM}{R^2}$.

Basic assumptions

Unless otherwise specified, we will assume the following:

- 1 **Ideal gas:** reasonable assumption for most regions of the star.

$pV_{\text{mol}} = \mathcal{R}T$, where V_{mol} is the molar volume (volume occupied by 1 mole of gas).

$\Rightarrow p = \rho \frac{kT}{\mu m_p}$, with μm_p the **mean molecular weight** (mass of a “typical” particle in m_p).

- 2 **Hydrostatic equilibrium:** $\nabla p = -\rho g$, where $g = \frac{GM}{R^2}$.

- 3 **Plane-parallel atmosphere:** the piece of atmosphere under consideration is a flat “slab”, such that the physical quantities are only a function of height from the surface of the slab: $f(\mathbf{r}) \rightarrow f(z)$.

Only valid if scale height of atmosphere $H \ll R$.

Not valid in red giants! (“photosphere” difficult to define here!)

Illustration: extent of the solar photosphere

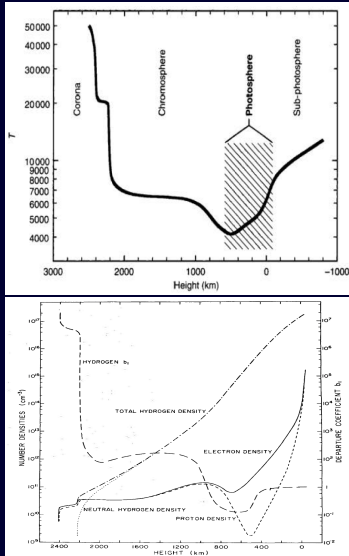
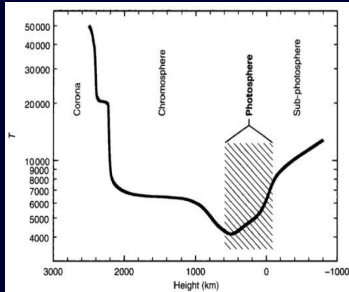


Figure on the bottom left confirms this exponential decay.

Illustration: extent of the solar photosphere



Over the photospheric region, ρ changes by $\sim 10^2$, T changes by $2\times \Rightarrow$ assume $T = \text{constant} = T_{\text{eff}}$.

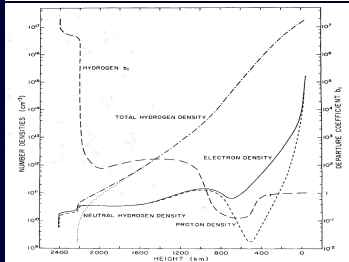
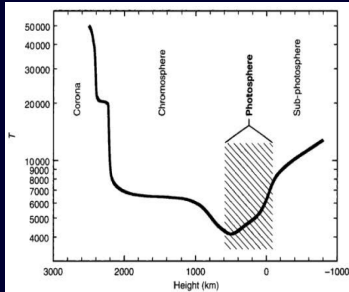


Figure on the bottom left confirms this exponential decay.

Illustration: extent of the solar photosphere



Over the photospheric region, ρ changes by $\sim 10^2$, T changes by $2\times \Rightarrow$ assume $T = \text{constant} = T_{\text{eff}}$.
 g also changes very slowly, as the enclosed mass is almost constant and $H \ll R$ (plane-parallel approx.)

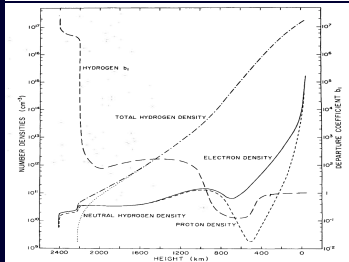
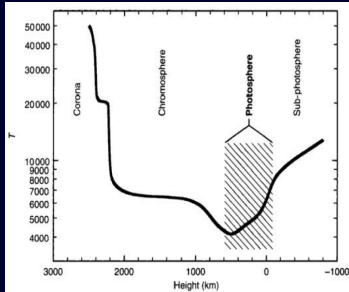


Figure on the bottom left confirms this exponential decay.

Illustration: extent of the solar photosphere



Over the photospheric region, ρ changes by $\sim 10^2$, T changes by $2\times \Rightarrow$ assume $T = \text{constant} = T_{\text{eff}}$.
 g also changes very slowly, as the enclosed mass is almost constant and $H \ll R$ (plane-parallel approx.)

The equation of hydrostatic equilibrium is then

$$\frac{d\rho}{dz} = -\rho g = -\rho \frac{GM}{R^2} = \frac{kT_{\text{eff}}}{\mu m_p} \frac{d\rho}{dz} \quad (\text{Ideal Gas Law}).$$

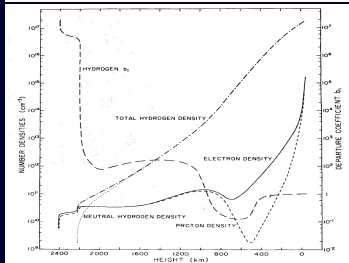
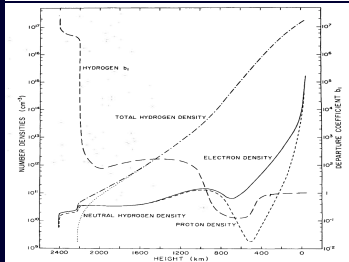
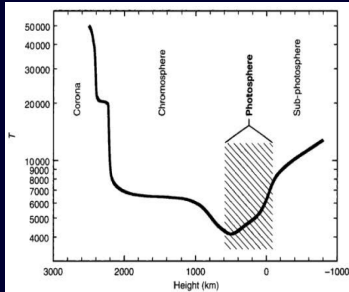


Figure on the bottom left confirms this exponential decay.

Illustration: extent of the solar photosphere



Over the photospheric region, ρ changes by $\sim 10^2$, T changes by $2\times \Rightarrow$ assume $T = \text{constant} = T_{\text{eff}}$.
 g also changes very slowly, as the enclosed mass is almost constant and $H \ll R$ (plane-parallel approx.)

The equation of hydrostatic equilibrium is then

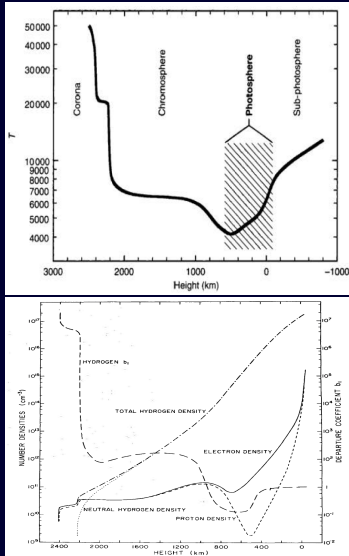
$$\frac{dp}{dz} = -\rho g = -\rho \frac{GM}{R^2} = \frac{kT_{\text{eff}}}{\mu m_p} \frac{d\rho}{dz} \quad (\text{Ideal Gas Law}).$$

$$\Rightarrow \rho(z) = \rho(z=0) \exp\left[-\frac{z}{H}\right],$$

$$\text{with scale height } H = \frac{kT_{\text{eff}}}{\mu m_p} \frac{R^2}{GM}.$$

Figure on the bottom left confirms this exponential decay.

Illustration: extent of the solar photosphere



Over the photospheric region, ρ changes by $\sim 10^2$, T changes by $2\times \Rightarrow$ assume $T = \text{constant} = T_{\text{eff}}$.
 g also changes very slowly, as the enclosed mass is almost constant and $H \ll R$ (plane-parallel approx.)

The equation of hydrostatic equilibrium is then

$$\frac{dp}{dz} = -\rho g = -\rho \frac{GM}{R^2} = \frac{kT_{\text{eff}}}{\mu m_p} \frac{d\rho}{dz} \quad (\text{Ideal Gas Law}).$$

$$\Rightarrow \rho(z) = \rho(z=0) \exp\left[-\frac{z}{H}\right],$$

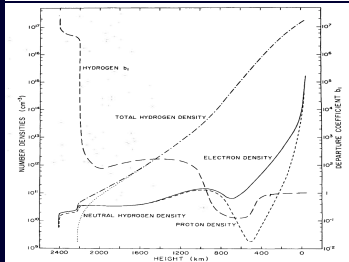
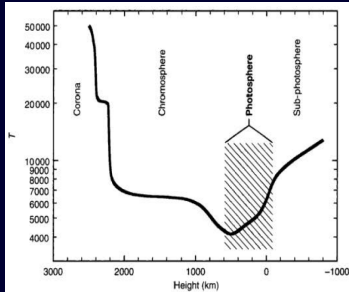
$$\text{with scale height } H = \frac{kT_{\text{eff}}}{\mu m_p} \frac{R^2}{GM}.$$

Figure on the bottom left confirms this exponential decay.

Substitute for R using the Stefan-Boltzmann Law,

$$H = \frac{k}{GM\mu m_p} \frac{L}{4\pi\sigma T_{\text{eff}}^3} \approx 174 \text{ km, with } \mu = 1.$$

Illustration: extent of the solar photosphere



Over the photospheric region, ρ changes by $\sim 10^2$, T changes by $2\times \Rightarrow$ assume $T = \text{constant} = T_{\text{eff}}$.
 g also changes very slowly, as the enclosed mass is almost constant and $H \ll R$ (plane-parallel approx.)

The equation of hydrostatic equilibrium is then

$$\frac{dp}{dz} = -\rho g = -\rho \frac{GM}{R^2} = \frac{kT_{\text{eff}}}{\mu m_p} \frac{d\rho}{dz} \quad (\text{Ideal Gas Law}).$$

$$\Rightarrow \rho(z) = \rho(z=0) \exp\left[-\frac{z}{H}\right],$$

$$\text{with scale height } H = \frac{kT_{\text{eff}}}{\mu m_p} \frac{R^2}{GM}.$$

Figure on the bottom left confirms this exponential decay.

Substitute for R using the Stefan-Boltzmann Law,

$$H = \frac{k}{GM\mu m_p} \frac{L}{4\pi\sigma T_{\text{eff}}^3} \approx 174 \text{ km, with } \mu = 1.$$

T_{eff} and g have the dominant effect on nature of the atmosphere.

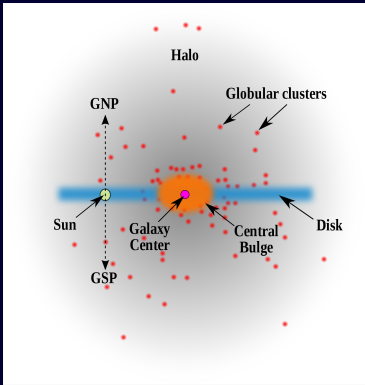
Chemical composition

The spectral features also depend on the chemical composition of the star.

Chemical composition

The spectral features also depend on the chemical composition of the star.

Metallicity: mass fraction of elements heavier than He (“metals”).



Credit: user:RJHall [CC BY-SA 3.0](#),
via Wikimedia Commons.

Chemical composition

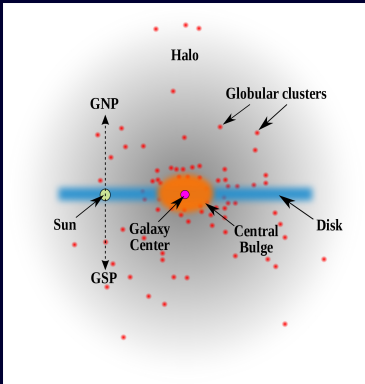
The spectral features also depend on the chemical composition of the star.

Metallicity: mass fraction of elements heavier than He (“metals”).

Hydrogen: $X \approx 0.7$

Helium: $Y \approx 0.29 \Rightarrow$ by number, $y_{He} \approx 0.08$

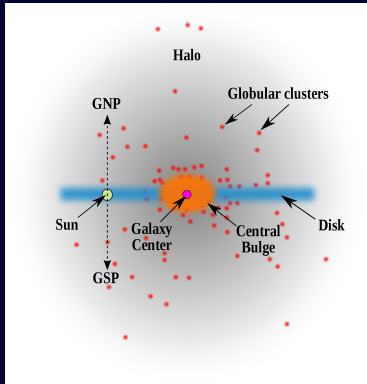
Everything else: $Z \approx 0.01$



Credit: user:RJHall [CC BY-SA 3.0](#),
via Wikimedia Commons.

Chemical composition

The spectral features also depend on the chemical composition of the star.



Credit: user:RJHall [CC BY-SA 3.0](#),
via Wikimedia Commons.

Metallicity: mass fraction of elements heavier than He (“metals”).

Hydrogen: $X \approx 0.7$

Helium: $Y \approx 0.29 \Rightarrow$ by number, $y_{\text{He}} \approx 0.08$

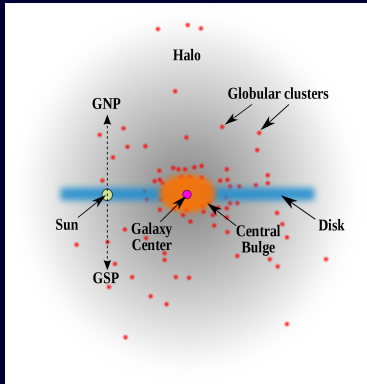
Everything else: $Z \approx 0.01$

CNO $Z \approx 10^{-4}$, $12 + \log \left(\frac{A}{H} \right) \approx 8.0$,

Fe $Z \approx 10^{-6}$

Chemical composition

The spectral features also depend on the chemical composition of the star.



Credit: user:RJHall [CC BY-SA 3.0](#),
via Wikimedia Commons.

Metallicity: mass fraction of elements heavier than He (“metals”).

Hydrogen: $X \approx 0.7$

Helium: $Y \approx 0.29 \Rightarrow$ by number, $y_{\text{He}} \approx 0.08$

Everything else: $Z \approx 0.01$

$$\text{CNO } Z \approx 10^{-4}, 12 + \log \left(\frac{A}{H} \right) \approx 8.0,$$

$$\text{Fe } Z \approx 10^{-6}$$

MW stars in two populations:

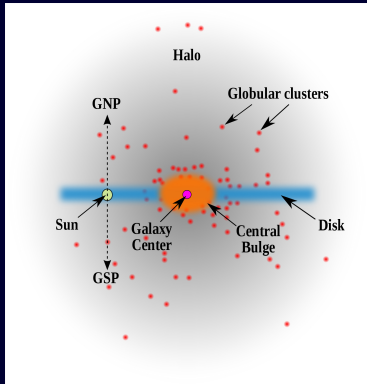
Population I: restricted to the disk. Higher Z . Sun-like and higher. $Z_{\odot} \approx 0.019$

Population II: Galactic halo. Lower Z . Sun-like and higher. $Z_{\odot} \approx 0.019$

Population III: “Primordial”, very low Z .

Chemical composition

The spectral features also depend on the chemical composition of the star.



Credit: user:RJHall [CC BY-SA 3.0](#),
via Wikimedia Commons.

Metallicity: mass fraction of elements heavier than He (“metals”).

Hydrogen: $X \approx 0.7$

Helium: $Y \approx 0.29 \Rightarrow$ by number, $y_{\text{He}} \approx 0.08$

Everything else: $Z \approx 0.01$

$$\text{CNO } Z \approx 10^{-4}, 12 + \log \left(\frac{A}{H} \right) \approx 8.0,$$

$$\text{Fe } Z \approx 10^{-6}$$

MW stars in two populations:

Population I: restricted to the disk. Higher Z . Sun-like and higher. $Z_{\odot} \approx 0.019$

Population II: Galactic halo. Lower Z . Sun-like and higher. $Z_{\odot} \approx 0.019$

Population III: “Primordial”, very low Z .

Chemical composition also evolves as nucleosynthetic products are brought out to the surface (convection, rotation).

Velocity and speed distribution

In thermal equilibrium at temperature T , the velocities of gas particles of mass m are distributed according to a Gaussian distribution:

$$dP(\mathbf{v}) = \text{fraction of particles in with velocities in } (\mathbf{v}, \mathbf{v} + d\mathbf{v}) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left[-\frac{mv^2}{2kT}\right] d\mathbf{v}.$$

Velocity and speed distribution

In thermal equilibrium at temperature T , the velocities of gas particles of mass m are distributed according to a Gaussian distribution:

$$dP(\mathbf{v}) = \text{fraction of particles in with velocities in } (\mathbf{v}, \mathbf{v} + d\mathbf{v}) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left[-\frac{mv^2}{2kT}\right] d\mathbf{v}.$$

Spectral line with natural frequency ν_0 from source with radial velocity v_R Doppler shifted by

$$\frac{\Delta\nu}{\nu_0} = \frac{v_R}{c}.$$

Thermal distribution of v_R : $dP(v_R) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left[-\frac{mv_R^2}{2kT}\right] dv_R.$

Velocity and speed distribution

In thermal equilibrium at temperature T , the velocities of gas particles of mass m are distributed according to a Gaussian distribution:

$$dP(\mathbf{v}) = \text{fraction of particles in with velocities in } (\mathbf{v}, \mathbf{v} + d\mathbf{v}) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left[-\frac{mv^2}{2kT}\right] d\mathbf{v}.$$

Spectral line with natural frequency ν_0 from source with radial velocity v_R **Doppler shifted** by

$$\frac{\Delta\nu}{\nu_0} = \frac{v_R}{c}.$$

Thermal distribution of v_R : $dP(v_R) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left[-\frac{mv_R^2}{2kT}\right] dv_R.$

⇒ **thermal component** to Doppler broadening of spectral lines.

Velocity and speed distribution

In thermal equilibrium at temperature T , the velocities of gas particles of mass m are distributed according to a Gaussian distribution:

$$dP(\mathbf{v}) = \text{fraction of particles in with velocities in } (\mathbf{v}, \mathbf{v} + d\mathbf{v}) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left[-\frac{mv^2}{2kT}\right] d\mathbf{v}.$$

Spectral line with natural frequency ν_0 from source with radial velocity v_R **Doppler shifted** by

$$\frac{\Delta\nu}{\nu_0} = \frac{v_R}{c}.$$

Thermal distribution of v_R : $dP(v_R) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left[-\frac{mv_R^2}{2kT}\right] dv_R.$

⇒ **thermal component** to Doppler broadening of spectral lines.

The volume element $d\mathbf{v} = dv_x dv_y dv_z$ (Cartesian) and $= 4\pi v^2 dv$ (Spherical).

Velocity and speed distribution

In thermal equilibrium at temperature T , the velocities of gas particles of mass m are distributed according to a Gaussian distribution:

$$dP(\mathbf{v}) = \text{fraction of particles in with velocities in } (\mathbf{v}, \mathbf{v} + d\mathbf{v}) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left[-\frac{m\mathbf{v}^2}{2kT}\right] d\mathbf{v}.$$

Spectral line with natural frequency ν_0 from source with radial velocity v_R **Doppler shifted** by

$$\frac{\Delta\nu}{\nu_0} = \frac{v_R}{c}.$$

$$\text{Thermal distribution of } v_R: dP(v_R) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left[-\frac{mv_R^2}{2kT}\right] dv_R.$$

⇒ **thermal component** to Doppler broadening of spectral lines.

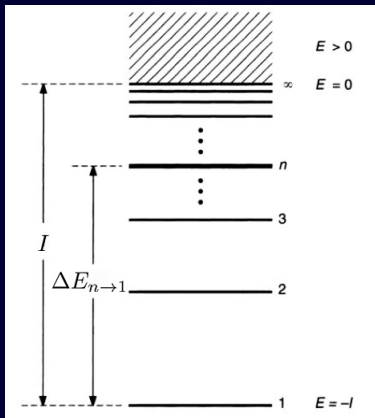
The volume element $d\mathbf{v} = dv_x dv_y dv_z$ (Cartesian) and $= 4\pi v^2 dv$ (Spherical).

The spherical choice gives us the distribution of **speeds**, the **Maxwell-Boltzmann Distribution**:

$$dP(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left[-\frac{mv^2}{2kT}\right] 4\pi v^2 dv.$$

Excitation

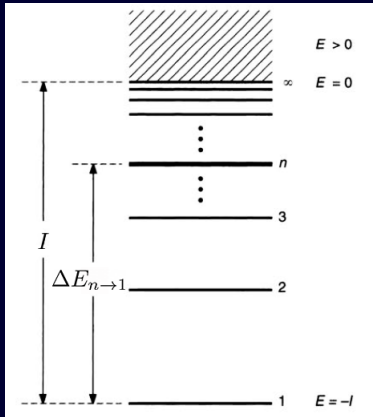
Electrons arranged in energy levels $n = 1, 2, 3, \dots$.



Excitation

Electrons arranged in energy levels $n = 1, 2, 3, \dots$.

Excitation energy = $\Delta E_{n_1 \rightarrow n_2} = E_{n_2} - E_{n_1}$
= Energy liberated upon de-excitation.

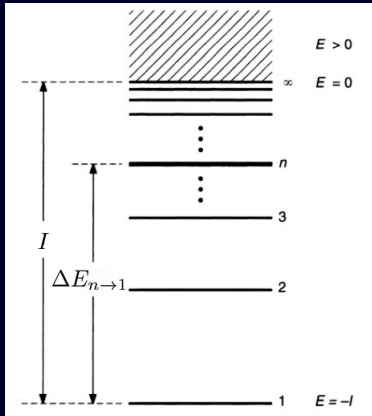


Excitation

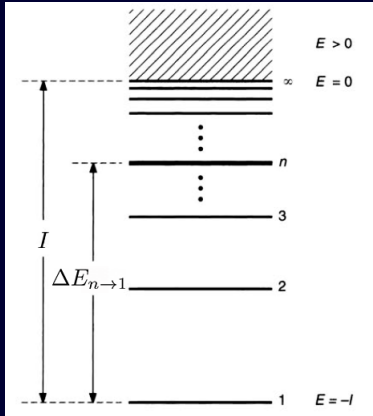
Electrons arranged in energy levels $n = 1, 2, 3, \dots$.

Excitation energy = $\Delta E_{n_1 \rightarrow n_2} = E_{n_2} - E_{n_1}$
= Energy liberated upon de-excitation.

(1st) Ionisation potential = $\Delta E_{\infty \rightarrow 1} \equiv I$



Excitation



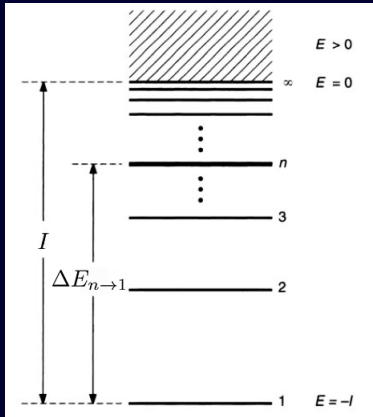
Electrons arranged in energy levels $n = 1, 2, 3, \dots$.

Excitation energy = $\Delta E_{n_1 \rightarrow n_2} = E_{n_2} - E_{n_1}$
= Energy liberated upon de-excitation.

(1st) Ionisation potential = $\Delta E_{\infty \rightarrow 1} \equiv I$

$\Rightarrow E_{\text{ground state}} = -I$ w.r.t. "free" electron. $E_n \leq 0$ for $n = 1, 2, 3, \dots$ (bound states).

Excitation



Electrons arranged in energy levels $n = 1, 2, 3, \dots$.

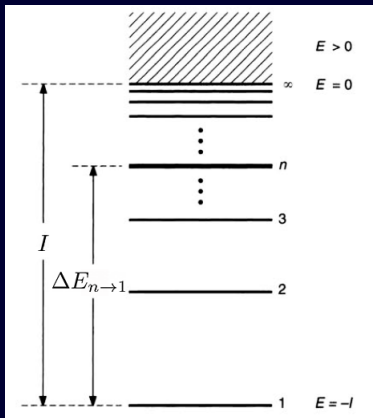
Excitation energy = $\Delta E_{n_1 \rightarrow n_2} = E_{n_2} - E_{n_1}$
= Energy liberated upon de-excitation.

(1st) Ionisation potential = $\Delta E_{\infty \rightarrow 1} \equiv I$

$\Rightarrow E_{\text{ground state}} = -I$ w.r.t. "free" electron. $E_n \leq 0$ for $n = 1, 2, 3, \dots$ (bound states).

Collisional excitation: At $T \neq 0$, always some atoms in excited states.

Excitation



Electrons arranged in energy levels $n = 1, 2, 3, \dots$.

Excitation energy = $\Delta E_{n_1 \rightarrow n_2} = E_{n_2} - E_{n_1}$
 = Energy liberated upon de-excitation.

(1st) Ionisation potential = $\Delta E_{\infty \rightarrow 1} \equiv I$

$\Rightarrow E_{\text{ground state}} = -I$ w.r.t. "free" electron. $E_n \leq 0$ for $n = 1, 2, 3, \dots$ (bound states).

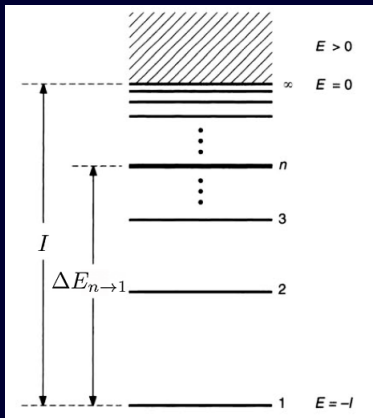
Collisional excitation: At $T \neq 0$, always some atoms in excited states.

Fraction in state n : $\frac{N_n}{N} = \frac{g_n}{u(T)} \exp\left[-\frac{E_n}{kT}\right]$.

N = total atoms, g_n = statistical weight of n^{th} level,

$u(T)$ = **partition function** = $\sum_{j=1}^{\infty} g_j \exp\left[-\frac{E_j}{kT}\right]$.

Excitation



Electrons arranged in energy levels $n = 1, 2, 3, \dots$.

Excitation energy = $\Delta E_{n_1 \rightarrow n_2} = E_{n_2} - E_{n_1}$
 = Energy liberated upon de-excitation.

(1st) Ionisation potential = $\Delta E_{\infty \rightarrow 1} \equiv I$

$\Rightarrow E_{\text{ground state}} = -I$ w.r.t. "free" electron. $E_n \leq 0$ for $n = 1, 2, 3, \dots$ (bound states).

Collisional excitation: At $T \neq 0$, always some atoms in excited states.

Fraction in state n : $\frac{N_n}{N} = \frac{g_n}{u(T)} \exp\left[-\frac{E_n}{kT}\right]$.

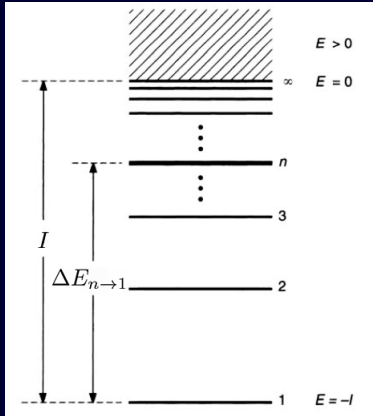
N = total atoms, g_n = statistical weight of n^{th} level,

$u(T)$ = **partition function** = $\sum_{j=1}^{\infty} g_j \exp\left[-\frac{E_j}{kT}\right]$.

Relative fraction in levels m and n ($m < n$):

$$\frac{N_n}{N_m} = \frac{g_n}{g_m} \exp\left[-\frac{\Delta E_{n \rightarrow m}}{kT}\right]$$

Excitation



Electrons arranged in energy levels $n = 1, 2, 3, \dots$.

Excitation energy = $\Delta E_{n_1 \rightarrow n_2} = E_{n_2} - E_{n_1}$
 = Energy liberated upon de-excitation.

(1st) Ionisation potential = $\Delta E_{\infty \rightarrow 1} \equiv I$

$\Rightarrow E_{\text{ground state}} = -I$ w.r.t. "free" electron. $E_n \leq 0$ for $n = 1, 2, 3, \dots$ (bound states).

Collisional excitation: At $T \neq 0$, always some atoms in excited states.

Fraction in state n : $\frac{N_n}{N} = \frac{g_n}{u(T)} \exp\left[-\frac{E_n}{kT}\right]$.

N = total atoms, g_n = statistical weight of n^{th} level,

$u(T)$ = **partition function** = $\sum_{j=1}^{\infty} g_j \exp\left[-\frac{E_j}{kT}\right]$.

Relative fraction in levels m and n ($m < n$):

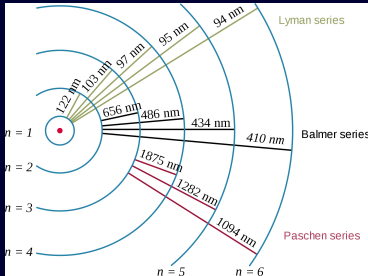
$$\frac{N_n}{N_m} = \frac{g_n}{g_m} \exp\left[-\frac{\Delta E_{n \rightarrow m}}{kT}\right]$$

For hydrogen atom, $g_n = 2n^2$.

Excitation - transitions in hydrogen-like atoms

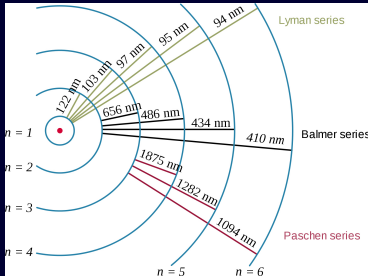
For light H-like (one-electron) atoms (H, He⁺),

Bohr model: $E_n = -13.6 \frac{Z^2}{n^2} \text{ eV}.$



Credit: user:OrangeDog [CC BY-SA 2.5](#),
via Wikimedia Commons.

Excitation - transitions in hydrogen-like atoms



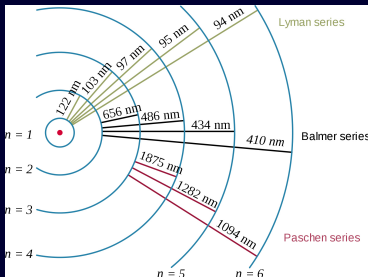
Credit: user:OrangeDog [CC BY-SA 2.5](#),
via Wikimedia Commons.

For light H-like (one-electron) atoms (H, He⁺),

Bohr model: $E_n = -13.6 \frac{Z^2}{n^2} \text{ eV}.$

$$\Rightarrow \Delta E_{n_2 \rightarrow n_1} = 13.6 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \text{ eV}.$$

Excitation - transitions in hydrogen-like atoms



Credit: user:OrangeDog [CC BY-SA 2.5](#),
via Wikimedia Commons.

For light H-like (one-electron) atoms (H, He⁺),

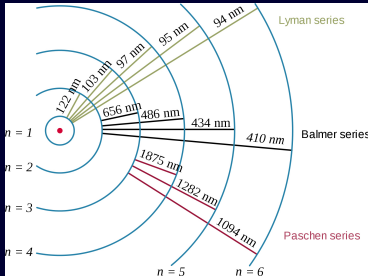
Bohr model: $E_n = -13.6 \frac{Z^2}{n^2} \text{ eV}.$

$$\Rightarrow \Delta E_{n_2 \rightarrow n_1} = 13.6 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \text{ eV}.$$

Transitions of H atom split into series based on n_1 :

$n_1 = 1$: Lyman series, $n_1 = 2$: Balmer, $n_1 = 3$: Paschen,
 $n_1 = 4$: Brackett, $n_1 = 5$: Pfund, $n_1 = 6$: Humphreys.

Excitation - transitions in hydrogen-like atoms



Credit: user:OrangeDog [CC BY-SA 2.5](#),
via Wikimedia Commons.

For light H-like (one-electron) atoms (H, He⁺),

Bohr model: $E_n = -13.6 \frac{Z^2}{n^2} \text{ eV}.$

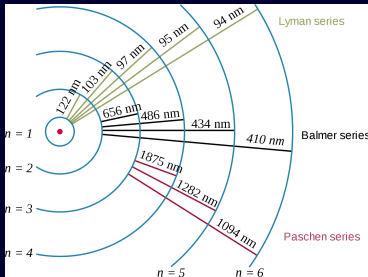
$$\Rightarrow \Delta E_{n_2 \rightarrow n_1} = 13.6 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \text{ eV}.$$

Transitions of H atom split into series based on n_1 :

$n_1 = 1$: Lyman series, $n_1 = 2$: Balmer, $n_1 = 3$: Paschen,
 $n_1 = 4$: Brackett, $n_1 = 5$: Pfund, $n_1 = 6$: Humphreys.

$n_2 = 2 \rightarrow n_1 = 1$: Lyman- α , $n_2 = 3 \rightarrow n_1 = 1$: Lyman- β ,
etc.

Excitation - transitions in hydrogen-like atoms



Credit: user:OrangeDog [CC BY-SA 2.5](#),
via Wikimedia Commons.

For light H-like (one-electron) atoms (H, He⁺),

Bohr model: $E_n = -13.6 \frac{Z^2}{n^2} \text{ eV}.$

$$\Rightarrow \Delta E_{n_2 \rightarrow n_1} = 13.6 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \text{ eV}.$$

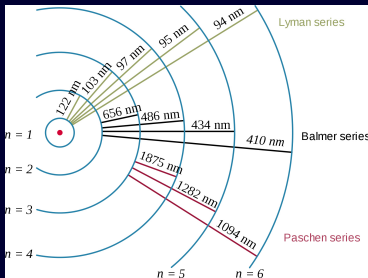
Transitions of H atom split into series based on n_1 :

$n_1 = 1$: Lyman series, $n_1 = 2$: Balmer, $n_1 = 3$: Paschen,
 $n_1 = 4$: Brackett, $n_1 = 5$: Pfund, $n_1 = 6$: Humphreys.

$n_2 = 2 \rightarrow n_1 = 1$: Lyman- α , $n_2 = 3 \rightarrow n_1 = 1$: Lyman- β ,
etc.

$n_2 = 3 \rightarrow n_1 = 2$: Balmer- α , also "H- α ", etc.

Excitation - transitions in hydrogen-like atoms



Credit: user:OrangeDog [CC BY-SA 2.5](#),
via Wikimedia Commons.

For light H-like (one-electron) atoms (H, He⁺),

Bohr model: $E_n = -13.6 \frac{Z^2}{n^2} \text{ eV}.$

$$\Rightarrow \Delta E_{n_2 \rightarrow n_1} = 13.6 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \text{ eV}.$$

Transitions of H atom split into series based on n_1 :

$n_1 = 1$: Lyman series, $n_1 = 2$: Balmer, $n_1 = 3$: Paschen,
 $n_1 = 4$: Brackett, $n_1 = 5$: Pfund, $n_1 = 6$: Humphreys.

$n_2 = 2 \rightarrow n_1 = 1$: Lyman- α , $n_2 = 3 \rightarrow n_1 = 1$: Lyman- β ,
etc.

$n_2 = 3 \rightarrow n_1 = 2$: Balmer- α , also "H- α ", etc.

Lyman series in UV/X-rays, Balmer in optical and
near-IR, others at longer wavelengths.