Stellar Atmospheres: Lecture 4, 2020.04.27

Prof. Sundar Srinivasan

IRyA/UNAM

Exam #1: Monday, 2020-05-04

Feel free to use your notes, books (I will supply the relevant chapters), and the internet.

Starts: 9 AM on Monday, 2020-05-04.

Due: 5 PM on Tuesday, 2020-05-05.

All scripts must be emailed in .py or .nb format, not as a PDF.

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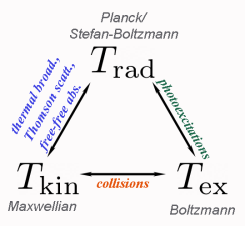

Adapted from Aaron Parson's YouTube clips here and here.

Poll: excitation temperature

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In LTE, $T_{\rm ex} = T_{\rm kin} = T_{\rm rad}$.

Radiative transfer

Sources:
Hubeny & Mihalas, chapters 3 and 11
Rybicki & Lightman, chapter 1
Aaron Parsons' lectures on YouTube.

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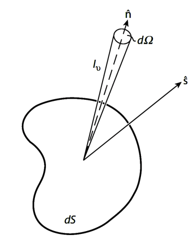

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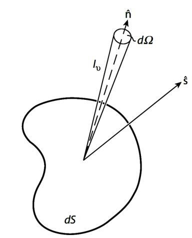

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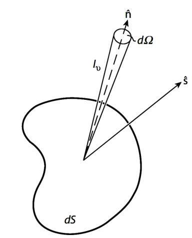

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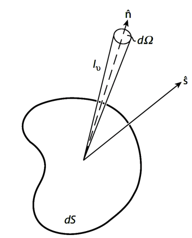

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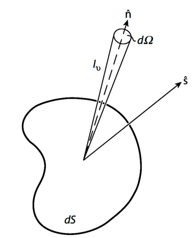

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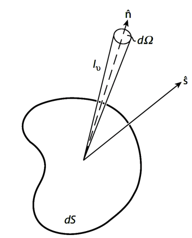

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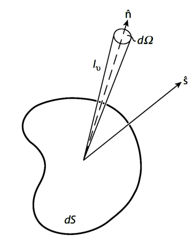

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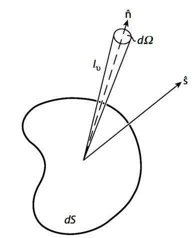

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Absorption + scattering = attenuation or extinction.

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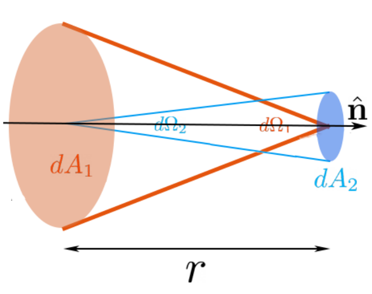

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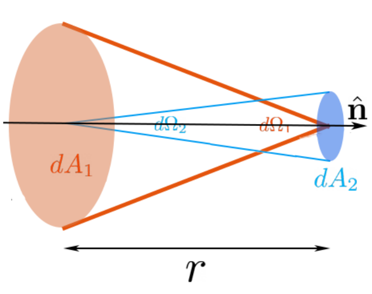

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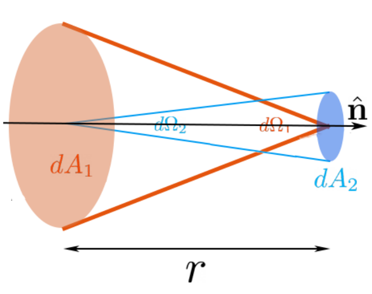

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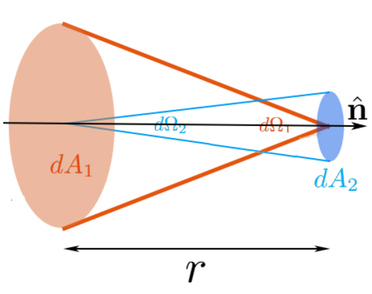

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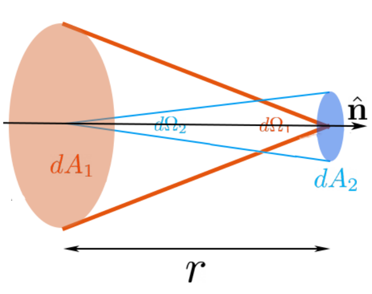

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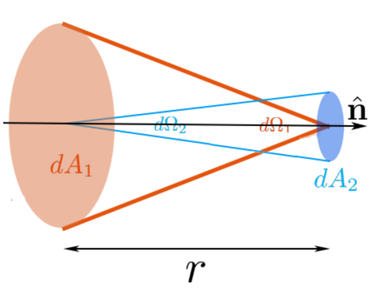

Thus, $\frac{dI_\nu}{dr} = 0$ (no absorption, emission, scattering). In the presence of absorption, emission, or scattering, $\frac{dI_\nu}{dr}$ is given by the radiative transfer equation.

The radiative transfer (RT) equation

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τ_ν is the optical depth (= fractional change in intensity for pure absorption). Measured along the direction of propagation (from source to observer).

For arbitrary s_0, $\tau(s_0, s) \equiv \int_{s_0}^{s} \alpha_\nu(s')\, ds'$, with $s_0 \geq s$. Note that $\tau(s_0, s_0) = 0$.

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The radiative transfer equation is highly nonlinear. Simplifying assumptions used for some standard solutions.

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For pure absorption,

$$\frac{dI_\nu}{ds} = -\alpha_\nu\ I_\nu \Longrightarrow I_\nu(s) = I_\nu(s_0)\ \exp\left[-\int\limits_{s_0}^{s} \alpha_\nu(s')\ ds'\right] \equiv I_\nu(s_0)\ e^{-\tau_\nu(s_0,s)}.$$

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The optical depth is thus the fractional reduction in intensity in the pure-absorption case.

$\tau < 1$: "optically thin" or "transparent" material. $\tau > 1$: "optically thick" or "opaque" material.

Poll: optical depth at the photosphere

RT equation: formal solution (no scattering)

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Multiply both sides by e^{τ_ν} and integrate from $\tau_\nu(s_0, s_0) = 0$ to $\tau_\nu(s_0, s)$ (called τ_ν for brevity):

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At any point on ray, if $I_\nu > S_\nu$, $\frac{dI_\nu}{d\tau_\nu} < 0$ so $I_\nu \downarrow$ along ray. If $I_\nu < S_\nu$, $I_\nu \uparrow$ along ray.

Eventually relaxes to S_ν in either case.

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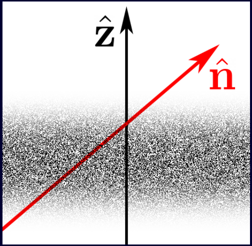

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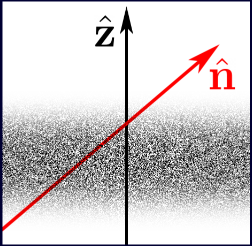

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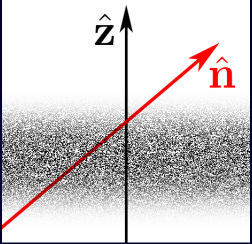

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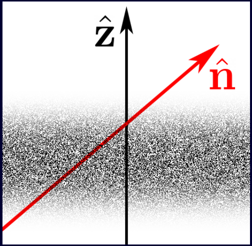

We can average over the μ dependence to obtain pure functions of ν, z.

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The first three moments of $I_\nu(\mu)$ are, therefore,

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Because of (1), J_ν is the mean intensity. Similarly, $H_\nu = 4\pi F_\nu$ where F_ν is the net flux along **n**, and $\frac{4\pi K_\nu}{c} = p_\nu$, the momentum flux along **n**.