Stellar Atmospheres: Lecture 6, 2020.05.11

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Review

Assumptions made to estimate properties of a "normal" stellar atmosphere:

1. LTE
2. Plane parallel atmosphere
3. Surface gravity constant in atmosphere
4. Atmospheric structure determined by continuum opacity only.
5. Radiation is the only source of energy

Due to these assumptions, we only require 3 parameters to specify atmospheric structure: g, $T_{\rm eff}$, and Z.

$\longrightarrow$ RT equation and its solution, $S_\nu(\tau_\nu)$, $T(\tau_\nu)$, etc...

Source of continuum and line opacities

References

- Hubeny & Mihalas Chapter 5;
- Collins, *The Fundamentals of Stellar Astrophysics*, Chapter 13, 14;
- Rybicki & Lightman Chapter 3;
- Gray Chapter 8, 11.

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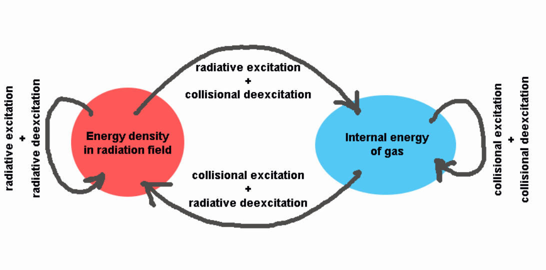
Processes may or may not involve coupling of radiation field with gas physics.

Bound-bound and bound-free emission/absorption



Replace: excitation $\longrightarrow$ ionisation, deexcitation $\longrightarrow$ recombination.

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Typically we fit a smooth function to the continuum and remove it (“continuum subtraction/division”) to analyse spectral lines (*e.g.*, computing an equivalent width).

Continuum identification/removal is one of the largest sources of error when studying spectral lines!

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- No net energy exchange with local thermal pool; however, emitted photon has different (random) direction. Travels a different distance through atmosphere and may be absorbed elsewhere. Analysis of resonance scattering line does not provide information about local conditions.

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RT equation: $\mu \dfrac{dI_\nu}{d\tau_\nu} = I_\nu - J_\nu$, with $d\tau_\nu \equiv -\rho\sigma^{\rm sca}_{\nu,{\rm line}} dz$

Form similar to that for Grey atm. $\Rightarrow F_\nu(\tau_\nu) =$ constant (monochromatic flux constancy).

Solution using 2-stream approximation (For details, see Collins, Ch. 13, pp. 336-339).

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The Milne-Eddington model allows for line absorption, but assumes no continuum scattering.

Line profile

Line transition: associated with a certain ΔE.

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Line profile: Probability distribution of a photon with ν around ν_0 being absorbed/emitted.

Normalisation:

$$\int \phi_\nu \, d\nu = 1 \text{ (indep. of direction)} \qquad \int d\Omega \int \phi_\nu(\hat{\mathbf{n}}) \, d\nu = 1 \text{ (dep. on direction)}$$

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Cross section for radiation emitted by electron:

$$\sigma(\omega) \equiv \frac{\text{time-avgd. radiative power}}{\text{time-avgd. intensity}} = \sigma_{\mathrm{T}} \frac{\omega^4}{(\omega^2 - \omega_0^2)^2 + (\Gamma\omega_0)^2}$$

$$\sigma_{\mathrm{T}} = \frac{8\pi}{3} r_e^2 = \text{Thomson Cross Section} \approx 66.53\ \text{fm}^2.$$

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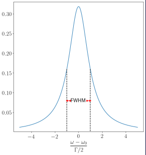

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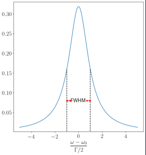

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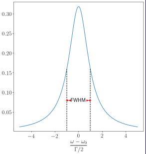

Generalise: $\int_0^\infty \sigma(\omega) d\omega \equiv \sigma_L\, f$,

with f = oscillator strength $\propto$ transition probability, “equiv. # of classical oscillators”.

Electric dipole: $f \sim 1$; electric quadrupole/magnetic dipole (“forbidden”): $f \ll 1$.

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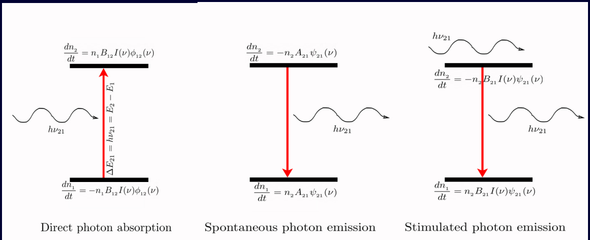

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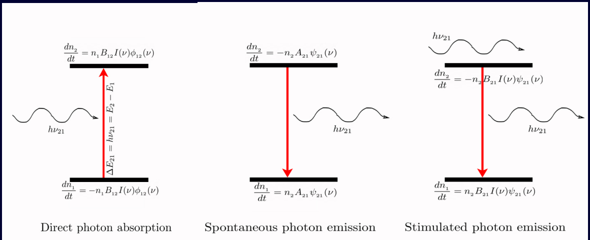

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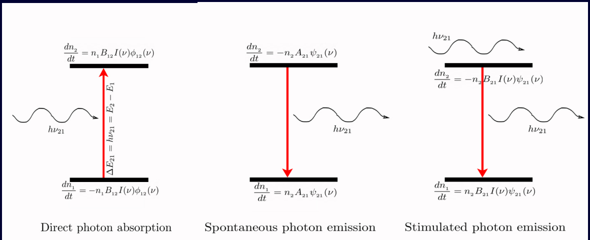

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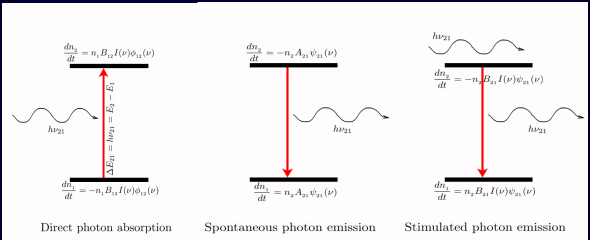

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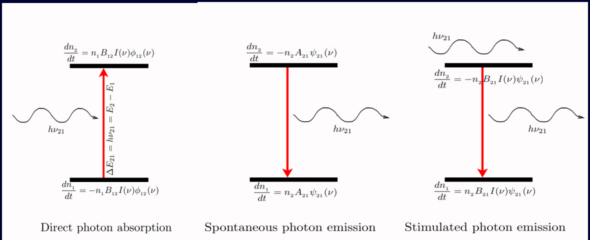

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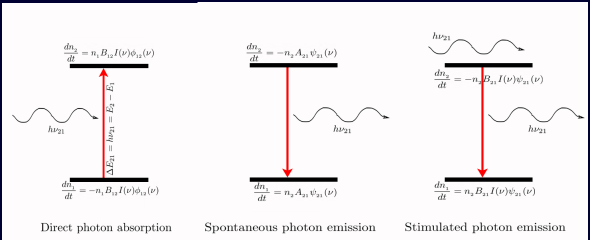

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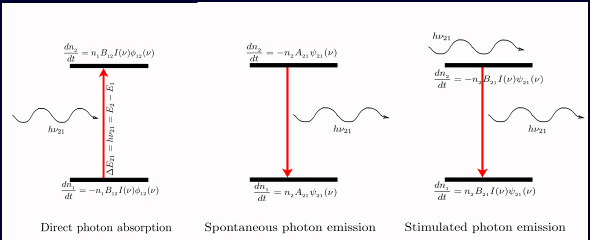

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Determining the actual values of the coefficients requires quantum mechanics. We can also relate them to scattering coefficients and lifetimes.

Analysis extended to bound-free transitions – Milne Relations; see Hubeny & Mihalas, Ch. 5, pp 119-121.

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Assuming $\phi_{12}(\nu) \approx \delta(\nu - \nu_{21})$, $\int \sigma_\nu d\nu \approx B_{12} \frac{h\nu_{21}}{4\pi} \left(1 - \exp\left[-\frac{h\nu_{12}}{kT}\right]\right) \equiv f_{12}\sigma_L$.

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Electronic orbitals: 0.1 eV (NIR) $\rightarrow$ 1-10 eV (optical) $\rightarrow$ 100 eV (UV).

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Prohibited lines: E2 ($\Delta L = 2$), M1 ($\Delta L = 0$). $A_{21} \sim 1 - 100$ s$^{-1}, t_{\rm rad} \sim 10$ ms – 1 s.

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Complication: systems with multiple intermediate (metastable) levels – electron may deexcite in stages (fluorescence), or partly via collisions and then to ground state (Raman Scattering).