Stellar Atmospheres: Lecture 8, 2020.05.15

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IRyA/UNAM

Continuum opacities

References:

- Hubeny & Mihalas Chapter 5;
- Collins, *The Fundamentals of Stellar Astrophysics*, Chapter 13, 14;
- Rybicki & Lightman Chapter 3;
- Gray Chapter 8, 11.
- Aller Chapter 5, 6.

Sources of continuum opacity

- Free-free transitions (Bremsstrahlung)
- Bound-free transitions (photoionisation)
- Hydride ion photodissociation
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Hydrogen-related processes dominant source of absorption for B–K stars.

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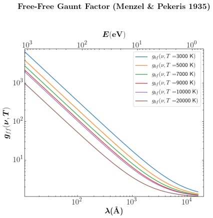

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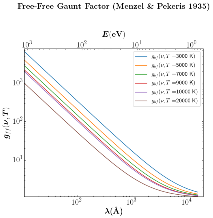

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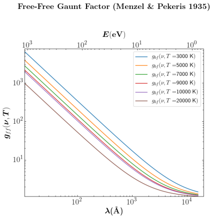

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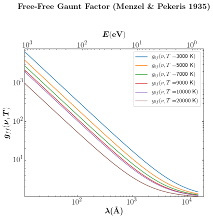

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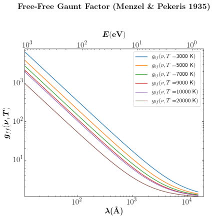

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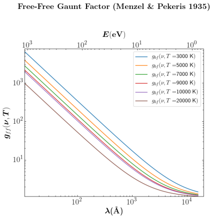

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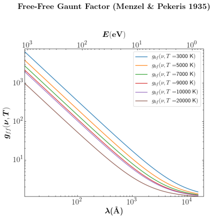

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Integrate over frequency:

$$j^{ff} = 4\frac{e^2}{6\pi\epsilon_0 c^3}\left(\frac{Ze^2}{4\pi\epsilon_0 m_e}\right)^2\left(\frac{2\pi m_e kT}{3h^2}\right)^{1/2} n_e n_i \overline{g_{ff}(T)} = 1.1\times 10^{-41} Z^2 \overbrace{n_e n_i}^{\text{SI units!}} T^{1/2}\ \text{W m}^{-3}; \qquad \overline{g_{ff}(T)} \sim 1.1\text{–}1.5.$$

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In Rayleigh-Jeans regime, approximate relation:

$$1-e^{-h\nu/kT} \approx \frac{h\nu}{kT} \implies \alpha_\nu^{ff} \approx 1.8\times 10^{-12}\ T^{-3/2}\ Z^2 \underbrace{n_e n_i}_{\text{SI units!}}\ \nu^{-2}\ \overline{g_{ff}(T,\nu)} \qquad (\text{units: m}^{-1}).$$

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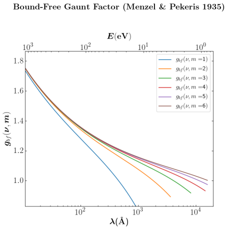

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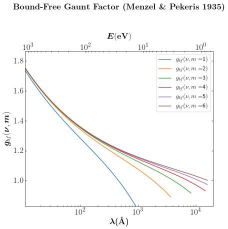

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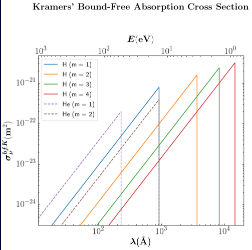

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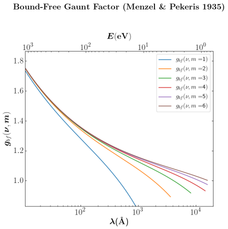
Hydrogen-like species: HI, HeII, CVI, OVIII, $\cdots$ tighter binding of e^- (energy levels $\propto Z^2$).

Example: $E(\mathrm{H}, m = 1) = E(\mathrm{He}, m = 2)$.



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For H $\rightarrow$ p + e, $g_{0,m} = 2m^2, g_e = 2, g_i = 1$. Use $h\nu = h\nu_m + \frac{1}{2} m_e v^2$ and $h\nu_m = \frac{m_e c^2}{2m^2} \alpha^2 Z^2$:

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For $\nu \gtrsim 10\nu_m$, $\sigma_\nu^{fb} \approx m^2 \sigma_\nu^{bf}$. Diverges near ν_m (electrons with almost zero energy are easily captured).

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Next up: opacity of H^-.