# Stellar Atmospheres: Lecture 8, 2020.05.18

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IRyA/UNAM

# References

- An Introduction to Modern Astrophysics - Carroll & Ostlie
- Hubeny & Mihalas Chapter 8;
- Collins, *The Fundamentals of Stellar Astrophysics*, Chapter 13, 14;
- Gray Chapter 8, 11;
- Böhm-Vitense Chapter 7.

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where $\frac{1}{\overline{\alpha_R}} \equiv \frac{\int_0^\infty \frac{1}{\alpha_\nu} \frac{\partial B_\nu}{\partial T} d\nu}{\int_0^\infty \frac{\partial B_\nu}{\partial T} d\nu}$ is the Rosseland Mean Opacity. $\overline{\kappa_R}$ can be similarly defined (multiply above definition by $\rho$).

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The Rosseland Mean is frequency independent. More relevant to stellar interiors.

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$\Longrightarrow$ dominant contribution only around $IP \approx kT$.

Effective behaviour of $\alpha_\nu$ is Kramer-like ($\propto T^{-1/2}$).

# Hydride

Free-free emission dominates at high $T$. As $T \downarrow$, He and then H become neutral and $n_e \downarrow$.

Bound-free contribution from metals $\downarrow$ despite their lower IPs because # energetic photons $\downarrow$.

The hydride ion is the main source of opacity in this temperature range.

Ground state: $1s^2$ (singlet). No bound excited state.
Binding energy of $2^{\rm nd}$ electron: 0.754 eV ($\lambda = 1.64\mu$m, $E/k = 8650$ K).

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Source of electrons for $H^-$: abundant metals with lower ionisation potentials than H:
Na (5.1 eV), Mg (7.6 eV), Al (6 eV), Si (8.2 eV), Fe (7.9 eV)

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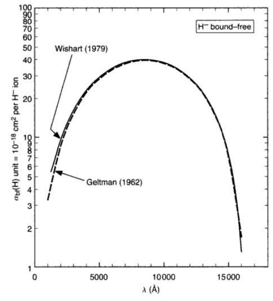


Gray Ch. 8 p. 155

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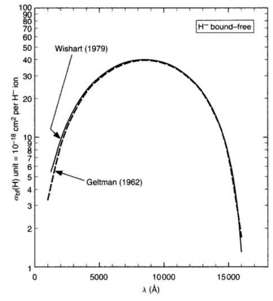


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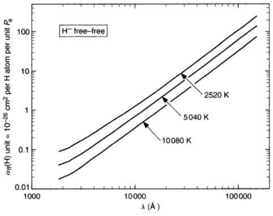


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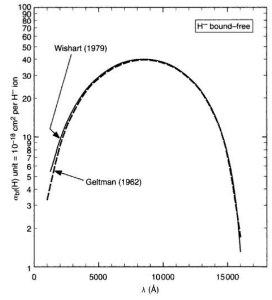


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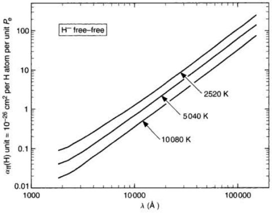


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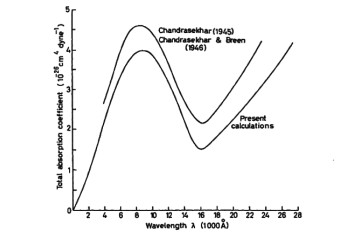


FIG. 2. *The absorption coefficient of the negative hydrogen ion at a temperature of 6 300 °K (θ=0·8) due to bound–free and free–free transitions. The combined results of Chandrasekhar (17) and Chandrasekhar & Breen (2) are included for comparison.*

Doughty & Fraser 1966 MNRAS 132 (255, 267)

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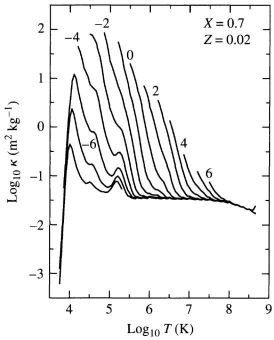


**FIGURE 9.10** Rosseland mean opacity for a composition that is 70% hydrogen, 28% helium, and 2% metals by mass. The curves are labeled by the logarithmic value of the density ($\log_{10}\rho$ in kg m$^{-3}$). (Data from Iglesias and Rogers, *Ap. J.*, *464*, 943, 1996.)

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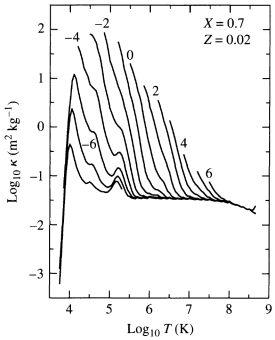


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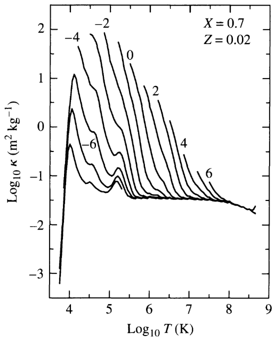


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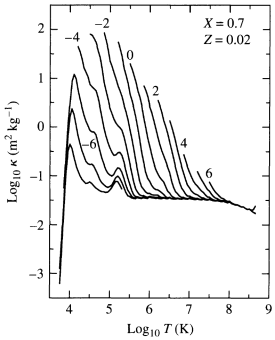


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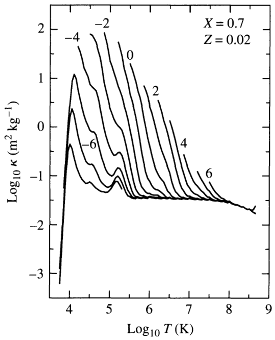


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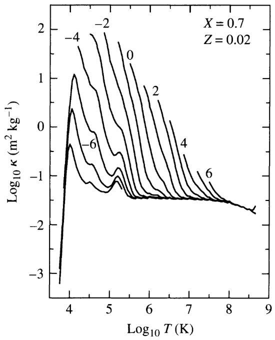


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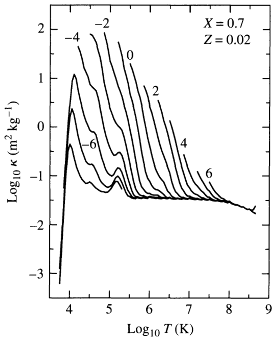


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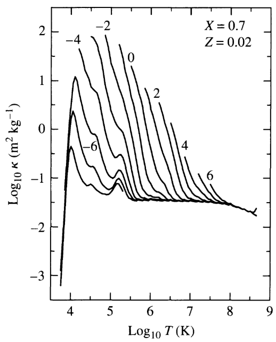


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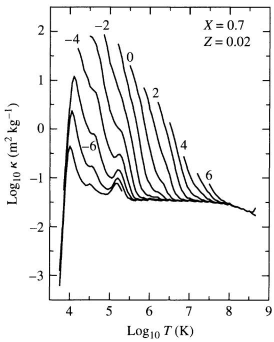


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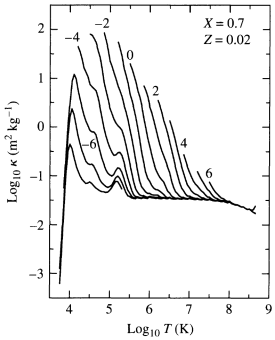


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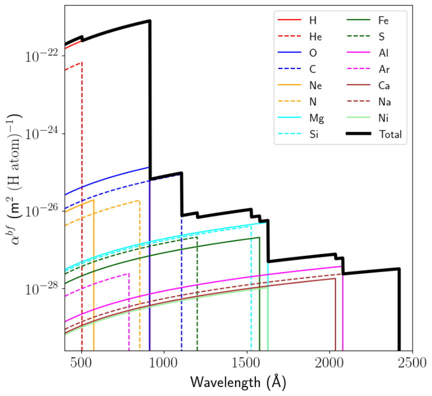
At even higher energies ($T \gtrsim 10^9$ K) Compton Scattering. Decrease in photon energy $\rightarrow$ opacity.

# Metals

Bound-free: dominant contribution from most abundant metals with IPs $\sim$ few eV.

(Plot only includes neutral species)




Rayleight scattering contribution non-negligible.

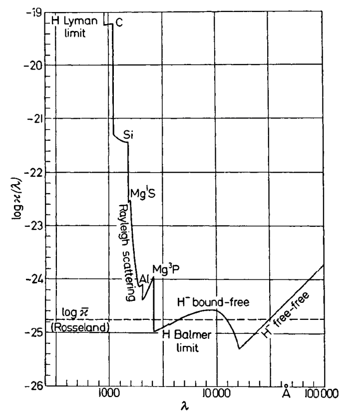



*Fig. 7.8.* The wavelength dependence of the continuous absorption coefficient per particle is shown for $T_{eff} = 5040$ K ($\Theta = 1.0$) and log $P_e = 0.5$.

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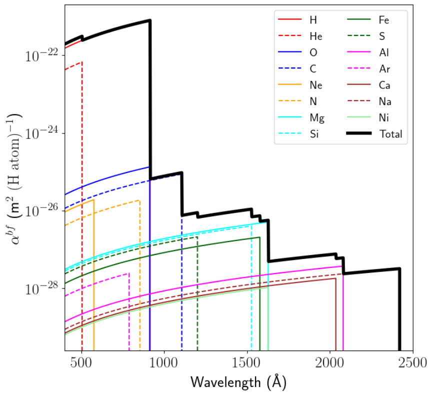
# Metals

Bound-free: dominant contribution from most abundant metals with IPs $\sim$ few eV.

(Plot only includes neutral species)

Note Lyman Jump.

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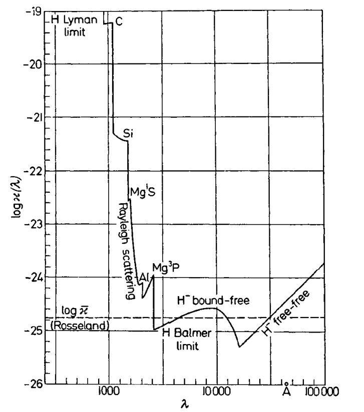


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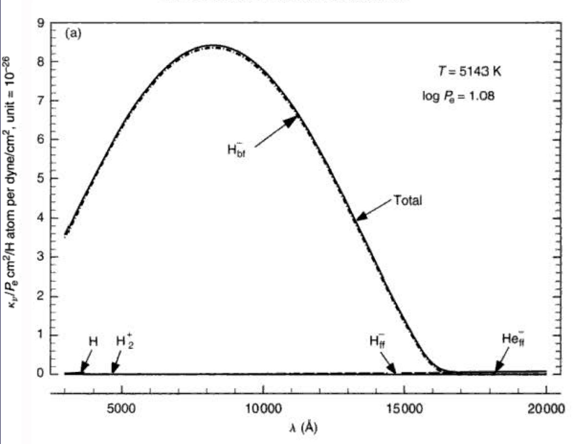
# Putting it all together




Gray Ch. 8 p. 160-162

At $T = 5413$ K, $H^-$ bound-free absorption dominates.

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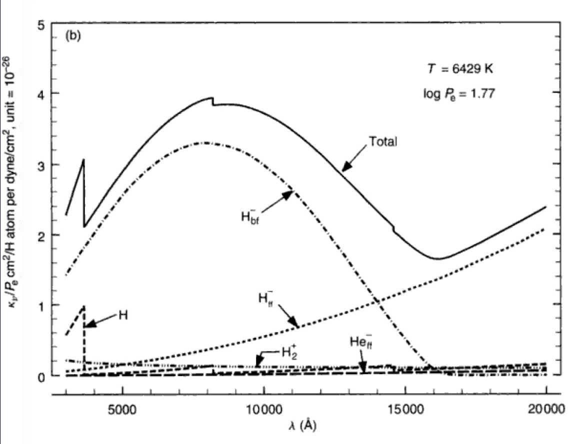


Gray Ch. 8 p. 160-162

At $T = 6429$ K,
$H^-$ bound-free and free-free absorption dominates.

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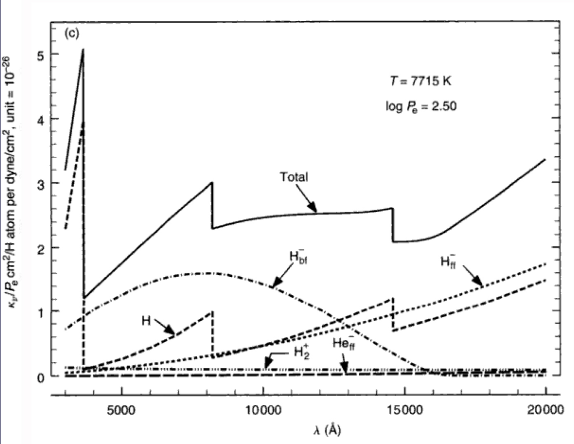


Gray Ch. 8 p. 160-162

At $T = 7715$ K,
HI contribution starts to increase.

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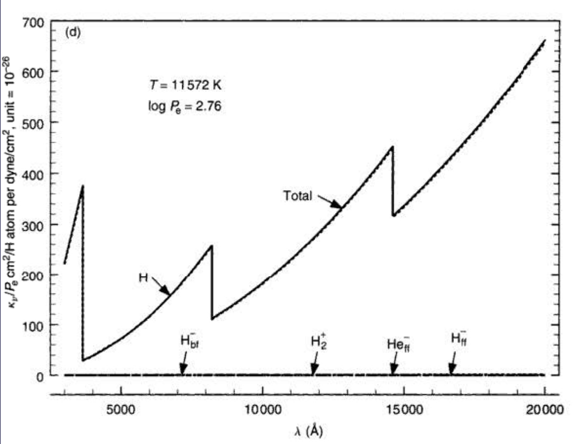


Gray Ch. 8 p. 160-162

At $T = 11752$ K,
HI dominates – significant increase in absorption.

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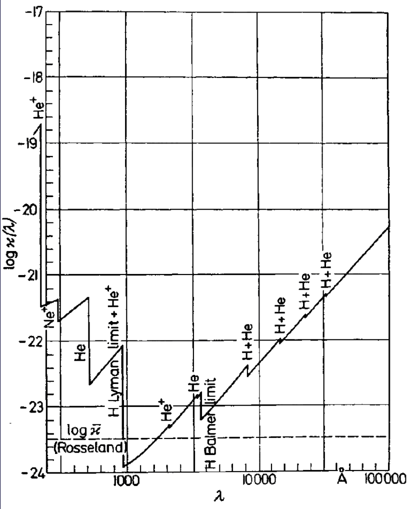


Böhm-Vitense Ch. 7, p. 85.

$T = 28300$ K (Main Seq. B0), $\log p_e = 2.5$

Mostly He contribution in the UV, H in the optical.

Electron scattering filling in at 1000–4000 Å.

# Line Broadening

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   Doesn't affect radiative transfer. Only enters into the calculation of observed flux.
   Rotation, macroturbulent (winds).

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Therefore, we expect a Lorentz Profile (see notes from Lecture 6).

$\sigma_{ij}(\nu) = \dfrac{4\pi^2 r_e c}{3} \dfrac{1}{(\Gamma/2)} \phi_L(x) f_{ij}$, with $f_{ij}$ the oscillator strength for the transition and $x = \dfrac{\nu - \nu_0}{(\Gamma/4\pi)}$.

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If $\sigma_{ij}(\nu_{\rm rest})$ naturally broadened, $\sigma_{ij}(\nu_{\rm obs})$ = convolution of Lorentz and Doppler profiles = Voigt Profile.

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Convolution of Lorentz (natural or pressure broadening) and Gaussian (Doppler, microturbulent) profiles.

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Shape: Gaussian centre with damping wings. Can be described by $H(a,u) \approx e^{-u^2} + \frac{a}{\sqrt{\pi} u^2}$