Stellar Atmospheres: Lecture 10, 2020.05.25

Prof. Sundar Srinivasan

IRyA/UNAM

References

1. Collins, *The Fundamentals of Stellar Astrophysics*, Ch 15.
2. Hubeny & Mihalas, Ch. 17
3. Gray, Ch 13.

Please download Lecture 9 again, fixed typos/omissions in the Eddington-Milne Equation slide.

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Line center: $\beta_\nu \gg 1 \Rightarrow \tau_c(\nu_0) \ll 1$. Lorentzian wings: $\beta_\nu \ll 1 \Rightarrow \tau_c \approx 2/3$. $\tau_c \in [0, 2/3]$ over entire range.

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In the wings, we can see deeper in the atmosphere, up to the $\tau_c = 2/3$ layer. Procedure for determining curve of growth:

1. compute β_ν for each ν.
2. compute τ_c using β_ν.
3. compute $S_\nu(\tau_c = 2/3/(1+\beta_\nu))$ and therefore the emergent line flux F_ν.
4. compute $S_\nu(\tau_c = 2/3)$ and therefore the emergent continuum flux F_c.
5. compute EQW.

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Populations no longer follow Maxwell-Boltzmann, Boltzmann, and Saha distributions.

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In TDE, (a) net flow must be zero (applicable for any time-independent state) (b) net flow out of individual levels must be zero. (b) $\rightarrow$ detailed balancing.

Consider a (bound/ionised) state labelled by i. The Continuity Equation is

$$\overbrace{\frac{dn_i}{dt}}^{\text{rate of change}} + \overbrace{\nabla \cdot (n_i \vec{u}_i)}^{\text{spatial gradient of particle flux}} = \overbrace{\sum_{j \neq i} [j \rightarrow i]}^{\text{transitions into } i} - \overbrace{\sum_{j \neq i} [i \rightarrow j]}^{\text{transitions out of } i} = 0 \text{ (rate equation for statistical equilibrium)}$$

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If we neglect advection due to particle flux, then we require # transitions into i = # transitions out of i.

Departure from LTE will affect both the line profile and the line strength.

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For radiative and collisional rates R and C, we have $\sum_{j\neq i} n_j(R_{ji} + n_e C_{ji}) = n_i \sum_{j\neq i}(R_{ij} + n_e C_{ij})$ Eqn. 1

In the above, $R_{ij} = A_{ij} + B_{ij} J_\nu$ if $i > j$ (deexcitation), and $R_{ij} = B_{ij} J_\nu$ if $i < j$ (excitation).
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Since the electrons deviate least from LTE, their speeds are still Maxwellian $\Rightarrow C_{ij}^* = C_{ij}$. Add the two equations to get

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LTE condition: $n_i = n_i^*$ for every $i \Rightarrow$ LHS of Eqn. 1 = LHS of Eqn. 2: $R_{ij} + n_e C_{ij} = R_{ij}^* + n_e C_{ij}$ for each ij combination.
If $\tau_\nu \to \infty$, $R_{ij} = R_{ij}^* \Rightarrow J_\nu = B_\nu$, we recover LTE.
At low τ_ν, the system must be collisionally dominated for LTE to apply: $[A_{ij}] + B_{ij}J_\nu \ll n_e \langle \sigma_e v_e \rangle_T$.
Possible either when T_{eff} low (J_ν low) or when g high (n_e high)
$\Rightarrow$ LTE valid in dwarfs, sort of in hot stars or giants. Definitely not valid in the chromosphere, corona, or in stellar winds.

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total energy levels $\to m$ rate equations of statistical equilibrium. Also need radiation field $\Rightarrow$ solution for RT equation.
RT solution must be consistent with stat. eq. solution. Can set up iterative procedure. Require NLTE source function.

Collisions vs. photoionisations in the Solar spectrum

Table 15.1 Types of Solar Spectral Lines

Collisionly Dominated	Dominated by Photoionization
Resonance lines of singly ionized metals	Resonance lines of neutral metals
Resonance lines of hydrogen	Balmer lines of hydrogen
Resonance lines of nonmetals	

Collins Ch. 15

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$$S_\nu \equiv (1 - a_\nu) \overbrace{B_\nu}^{\propto \text{ local conditions}} + \frac{a_\nu}{4\pi} \overbrace{\int_0^\infty \oint_{4\pi} R(\nu', \nu, \vec{\Omega}', \vec{\Omega}) I_{\nu'}(\vec{\Omega}') d\vec{\Omega}' d\nu}^{\propto \text{ radiation field only}}$$

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Isotropic scattering: $R(\nu', \nu, \vec{\Omega}', \vec{\Omega}) = g(\nu, \nu')$. Coherent scattering: $R(\nu', \nu, \vec{\Omega}', \vec{\Omega}) = h(\vec{\Omega}', \vec{\Omega})\delta(\nu - \nu')$.

Fully noncoherent scattering (scattered photon completely uncorrelated with incident photon): $R(\nu', \nu, \vec{\Omega}', \vec{\Omega}) = h(\vec{\Omega}', \vec{\Omega})$.

Coherent + isotropic scattering $\Rightarrow S_\nu = (1 - a_\nu)B_\nu + a\nu J_\nu$.

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$$S_\nu \equiv (1 - a_\nu) \overbrace{B_\nu}^{\propto \text{ local conditions}} + \overbrace{\frac{a_\nu}{4\pi} \int_0^\infty \oint_{4\pi} R(\nu', \nu, \vec{\Omega}', \vec{\Omega}) I_{\nu'}(\vec{\Omega}') d\vec{\Omega}' d\nu}^{\propto \text{ radiation field only}}$$

Isotropic scattering: $R(\nu', \nu, \vec{\Omega}', \vec{\Omega}) = g(\nu, \nu')$. Coherent scattering: $R(\nu', \nu, \vec{\Omega}', \vec{\Omega}) = h(\vec{\Omega}', \vec{\Omega})\delta(\nu - \nu')$.

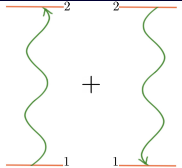
Fully noncoherent scattering (scattered photon completely uncorrelated with incident photon): $R(\nu', \nu, \vec{\Omega}', \vec{\Omega}) = h(\vec{\Omega}', \vec{\Omega})$.

Coherent + isotropic scattering $\Rightarrow S_\nu = (1 - a_\nu)B_\nu + a\nu J_\nu$.

If R independent of ν, ν' (as in fully noncoherent scattering) $\rightarrow$ complete redistribution.
Complete redistribution $\Longrightarrow$ a specific absorptive radiative transition is not correlated with a specific emissive radiative transition.

In what follows, we will assume complete redistribution.

Two-level atom - I



Emitted photon has different (random) direction $\rightarrow$ scattering.

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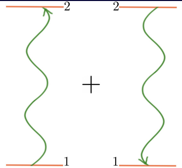

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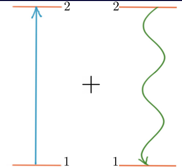

Emission produces thermal radiation.

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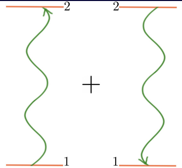

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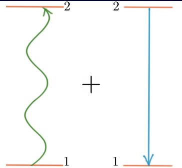
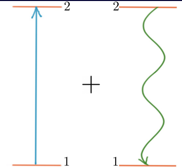


Absorption of photon into thermal pool of electrons.



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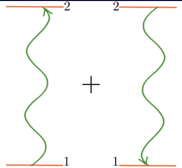

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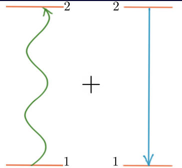

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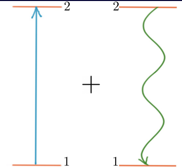

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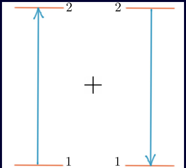
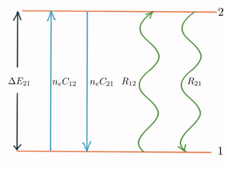


No effect on radiation.

Two-level atom - II



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Recall: $\dfrac{B_{21} g_2}{B_{12} g_1} = 1$ and $A_{21} = \dfrac{2h\nu^3}{c^2} \dfrac{g_1}{g_2} B_{12}$. Eqn. 1

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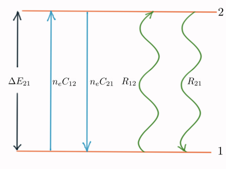

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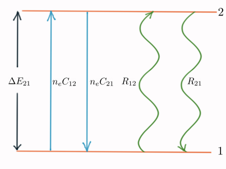

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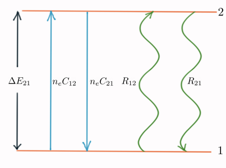

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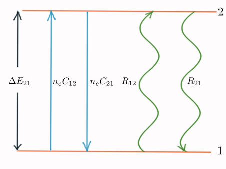

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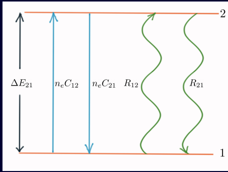

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Strong (resonance) lines: $A_{21} \gg C_{21}$ + formation high in the atmosphere (n_e small) $\Rightarrow$ severe deviation from LTE.

Line formation in the upper atmosphere

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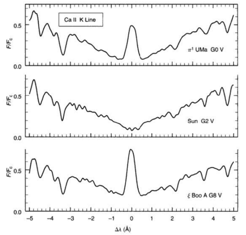
T first hits minimum above photosphere, then rises $\Rightarrow$ lines dip below continuum value, then rise in the centre.

This is the case for π^1 UMa and ξ Boo A.



CaII K lines centred at 3933 Å (Gray Ch 13)

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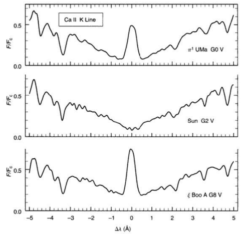

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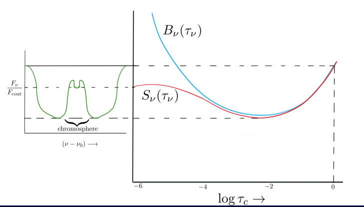

Line formation at low τ

Case 2. $\tau_\nu < 1$

n too low for LTE $\Rightarrow S_\nu < B_\nu$. Line centre shows a local minimum.

This is the case for the Sun (see figure above).