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Inverse Hubble flows in molecular clouds

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ABSTRACT

Motivated by recent numerical simulations of molecular cloud (MC) evolution, in which the clouds engage in global gravitational contraction, and local collapse events culminate significantly earlier than the global collapse, we investigate the growth of density perturbations embedded in a collapsing background, to which we refer as an *inverse Hubble flow* (IHF). We use the standard procedure for the growth of perturbations in a universe that first expands (the usual Hubble flow) and then recollapses (the IHF). We find that linear density perturbations immersed in an IHF grow faster than perturbations evolving in a static background (the standard Jeans analysis). A fundamental distinction between the two regimes is that, in the Jeans case, the time $\tau_{\rm nl}$ for a density fluctuation to become non-linear increases without limit as its initial value approaches zero, while in the IHF case $\tau_{\rm nl} \le \tau_{\rm ff}$ always, where $\tau_{\rm ff}$ is the free-fall time of the background density. We suggest that this effect, although moderate, implies that small-scale density fluctuations embedded in globally collapsing clouds must collapse earlier than their parent cloud, regardless of whether the initial amplitude of the fluctuations is moderate or strongly non-linear, thus allowing the classical mechanism of Hoyle fragmentation to operate in multi-Jeans-mass MCs. More fundamentally, our results show that, contrary to the standard paradigm that fluctuations of all scales grow at the same rate in the linear regime, the hierarchical nesting of the fluctuations of different scales does affect their growth even in the linear stage.

Key words: stars: formation – ISM: clouds – ISM: structure – galaxies: formation – large-scale structure of Universe.

1 INTRODUCTION

The fragmentation of a local overdensity (a 'cloud') in a continuum is one of the fundamental problems in astrophysics, as it underlies the formation of galaxy and star clusters. Over sixty years ago, Hoyle (1953) proposed a model in which stars form in a fragmentation process *during* the collapse of a nearly isothermal spherical cloud, based on the fact that the Jeans mass in an isothermal medium (or, more generally, in any polytropic medium with polytropic exponent $\gamma < 4/3$; e.g. Chandrasekhar 1961) decreases as the density increases. Hoyle's mechanism was subsequently laid on firmer mathematical grounds by Hunter (1962, 1964). This point of view prevailed until it was realized by Tohline (1980) that the small-scale density fluctuations within a cloud whose initial mass is close to the Jeans mass could not grow faster than the cloud itself, because when thermal pressure is non-negligible, it acts to slow down the collapse of smaller scales compared to the larger scales. Since then, it has been generally accepted that molecular clouds (MCs) do not fragment by a Hoyle-like mechanism, and other mechanisms, such as turbulent fragmentation (e.g. Mac Low & Klessen 2004) might be at work.

However, recent numerical studies suggest that cold, dense clouds in the interstellar medium (ISM) can form by intermediate- to large-scale converging flows in the warm atomic medium, which are capable of coherently triggering a phase transition to the cold atomic phase over large regions in the gas (e.g. Ballesteros-Paredes, Hartmann & Vázquez-Semadeni 1999; Hennebelle & Pérault 1999; Walder & Folini 2000; Hartmann, Ballesteros-Paredes & Bergin 2001; Koyama & Inutsuka 2002; Audit & Hennebelle 2005; Heitsch et al. 2005, 2006; Vázquez-Semadeni et al. 2006). This large-scale coherence implies that the clouds can form already containing a large number of Jeans masses (Vázquez-Semadeni et al. 2007). Therefore, the crucial assumption made by Tohline (1980), that the cloud's mass is near the Jeans mass, is not necessarily fulfilled in collapsing MCs, thus making it relevant to again consider the collapse of small-scale fluctuations within a larger-scale object which is itself collapsing, for the fragmentation of MCs.

The analysis of the collapse of density structures embedded within larger ones that are also undergoing collapse may benefit from the tools developed for studying the evolution of linear fluctuations in the cosmological flow of dark matter. In this case, it is

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standard to consider a pressureless, expanding Hubble flow, in which small-amplitude density enhancements at a certain scale L_0 begin to retard their expansion, until they eventually begin to collapse, at which point, they are said to 'separate' from the global expansion (e.g. Kolb & Turner 1990). It is well known that, in this case, the collapse of these regions proceeds more slowly (as a power-law in time) than that of a fluctuation in a non-expanding medium (which grows exponentially), because the global expansion counteracts the global collapse. Now, if being embedded in an expanding (regular Hubble) flow reduces the growth rate of the large-scale fluctuation, it is natural to expect that being embedded in a *contracting* (inverse Hubble) flow should *enhance* the growth rate of the fluctuations located inside the collapsing large-scale structure. This could be thought of as an inescapable form of non-linearity, in the sense that the growth of one mode is linked to that of another mode, since the growth of a small-scale fluctuation depends on its being located within a larger-scale one, an effect that will be active even when the small-scale fluctuation has a small (linear) amplitude.

In this paper, we investigate this possibility, using a linear perturbation analysis to study the growth of density fluctuations located inside a larger-scale spherical fluctuation undergoing free-fall collapse as well. Because, as discussed above, the MC case may be adequately described by means of a nearly pressureless regime, while the dark matter is intrinsically so, we consider a pressureless regime here. The solution for the density perturbation growth is then compared to that of the classical Jeans' case for a static background, to show that the initially linear density perturbations grow at a faster rate inside a collapsing spherical cloud.

2 THE GOVERNING EQUATIONS

2.1 The contracting background

In what follows, we will consider the collapse of a *spherical* cloud, embedded in a medium that is itself collapsing. Although MCs in particular are known to strongly depart from a spherical symmetry (e.g. Bally et al. 1987; Gutermuth et al. 2008; Myers 2009; Men'shchikov et al. 2010; Molinari et al. 2010), and the collapse times for non-spherical clouds are known to be longer than the standard free-fall time, which assumes this geometry (Pon et al. 2012; Toalá, Vázquez-Semadeni & Gómez 2012), the study of the spherical case will allow us to compare with this standard time-scale.

The collapse of a spherical cloud can be described using the standard machinery applied for a contracting Universe, noting that the physical differences lie in the definition of the Hubble parameter $H(t)$. In both cases this can be written as $H(t) = \dot{a}(t)/a(t)$, where $a(t)$ is a suitable scale factor. For the cosmological case, a is simply the well-known scale factor, while for a spherical MC, a can be defined as $a(t) = R(t)/R_0$, where R_0 is the initial cloud's radius. In both the MC and the cosmological cases, $a(t)$ satisfies Friedmann's equation (see Appendix A for the MC case)

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8}{3}\pi G\rho. \tag{1}$$

The case of a collapsing spherical cloud is mathematically equivalent to the second half of the evolution of a closed ($k > 0$), matter-dominated universe that first expands to a certain maximum scale factor and then recollapses. So, in what follows, we will consider that the origin of the time coordinate is the point of maximum expansion of such closed Universe. Note that, during this contracting phase, $\dot{a} < 0$.

2.2 The linear analysis

The standard linear stability analysis for the growth of density fluctuations in the case of an expanding (or contracting) universe starts from the linearized equations of continuity and momentum conservation, together with the Poisson equation, which read (e.g. Kolb & Turner 1990)

$$\frac{\partial\rho_1}{\partial t} + \frac{3\dot{a}}{a}\rho_1 + \frac{\dot{a}}{a}(\boldsymbol{r}\cdot\nabla)\rho_1 + \rho_0\nabla\cdot\boldsymbol{v}_1 = 0$$

$$\frac{\partial\boldsymbol{v}_1}{\partial t} + \frac{\dot{a}}{a}\boldsymbol{v}_1 + \frac{\dot{a}}{a}(\boldsymbol{r}\cdot\nabla)\boldsymbol{v}_1 + \frac{v_s^2}{\rho_0}\nabla\rho_1 + \nabla\varphi_1 = 0$$

$$\nabla^2\varphi_1 = 4\pi G\rho_1, \tag{2}$$

where v_s denotes the sound speed, $\boldsymbol{r}$ denotes the position vector, and the variables (generically denoted x) have been decomposed as $x = x_0 + x_1$, the subindex '0' denoting the unperturbed quantities, and the subindex '1' denoting the corresponding (small) fluctuation. The unperturbed quantities include the background expansion (or contraction)

$$\rho_0(t) = \rho_0(t_0)\,a^{-3}(t) \quad \boldsymbol{v}_0 = \frac{\dot{R}}{R}\boldsymbol{r} \quad \nabla\varphi_0 = \frac{4}{3}\pi G\rho_0\boldsymbol{r}, \tag{3}$$

where $a(t)$ is the scale factor, obeying the Friedmann equation, equation (1).

As is well known, for scales much larger than the Jeans scale, that is, neglecting the pressure term, the equation for the perturbation growth (e.g. Mukhanov 2005) is given by

$$\ddot{\delta} + 2H(t)\dot{\delta} - 4\pi G\rho_0\delta = 0, \tag{4}$$

where $\delta \equiv \rho_1/\rho_0 \equiv (\rho - \rho_0)/\rho_0$ is the relative density fluctuation. A general solution to equation (4) can be written as (Mukhanov 2005)

$$\delta = C_1 H(t)\int\frac{\mathrm{d}t}{a(t)^2H(t)^2} + C_2H(t), \tag{5}$$

where C_1 and C_2 are constants.

To find the solution, a parametrization is usually proposed in which both the scale factor a and time t are functions of a parameter θ. A usual parametrization is

$$t \propto (\theta - \sin\theta) \tag{6}$$

and

$$a(t) \propto (1 - \cos\theta), \tag{7}$$

with $\theta \in [0 : 2\pi]$ (Narlikar 1993). In Fig. 1 we have plotted the time and scale factor as a function of θ in normalized units for the collapsing part of the evolution in this model – that is, for $\theta \in [\pi : 2\pi]$. This case has been studied by Groth & Peebles (1975), who found the solution for δ as a function of θ as (see also Narlikar 1993)

$$\delta(\theta) = A\left[\frac{5+\cos\theta}{1-\cos\theta} - \frac{3\theta\sin\theta}{(1-\cos\theta)^2}\right] + B\frac{\sin\theta}{(1-\cos\theta)^2}, \tag{8}$$

where A and B are constants, which can be evaluated using the initial conditions as follows. We note that

$$\delta_{\theta=\pi} = 2A \tag{9}$$

and

$$\left(\frac{\mathrm{d}\delta}{\mathrm{d}\theta}\right)_{\theta=\pi} = \frac{3\pi A - B}{4}. \tag{10}$$

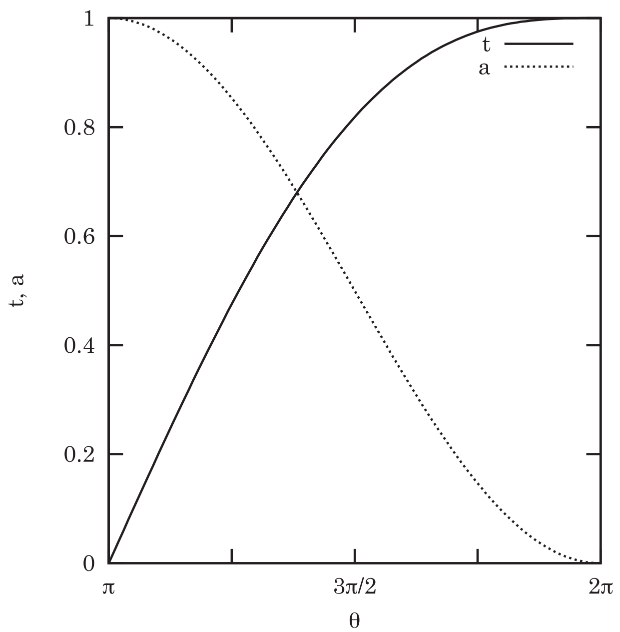

Figure 1. Normalized time t and scale factor a as a function of θ for the collapsing part ($\theta \in [\pi : 2\pi]$) of the closed matter-dominated universe.

For $B > 0$, the last term in equation (8) is positive but monotonically decreasing in the interval $0 < \theta < \pi$, approaching $+\infty$ as $\theta \to 0+$, and becoming zero at $\theta = \pi$. For $B < 0$, the signs are reversed, but the divergence of this term as $\theta \to 0$ persists. For $\pi < \theta < 2\pi$, it approaches ∞ as $\theta \to 2\pi$. On the other hand, the first term in the right-hand side of equation (8) is zero at $\theta = 0$. This implies that the full solution with $B \neq 0$ diverges there. For the cosmological case, which starts contracting at $\theta = \pi$, but must have previously undergone an expanding stage ($0 < \theta < \pi$), we must require that $\delta \to 0$ as $\theta \to 0+$ (the big bang), and thus we must take $B = 0$. The constant A is determined by equation (9), which gives $A = \delta_{\theta=\pi}/2$. Defining $\delta_{\theta=\pi} \equiv \delta_0 \equiv \rho_1(t=0)/\rho_0$, we can then write the evolution equation for the cosmological case as

$$\delta_{\mathrm{C}}(\theta) = \frac{\delta_0}{2}\left[\frac{5+\cos\theta}{1-\cos\theta} - \frac{3\theta\sin\theta}{(1-\cos\theta)^2}\right], \tag{11}$$

together with the initial condition, from equation (10),

$$\dot{\delta_{\mathrm{C}}}\big|_{\theta=\pi} = \frac{3\pi\delta_0}{8}, \tag{12}$$

where we have defined $\dot{\delta} \equiv \mathrm{d}\delta/\mathrm{d}\theta$.

On the other hand, for the MC case, for which there is no constraint that our initial state ($\theta = \pi$) be the continuation of a previous expanding stage, we have no boundary condition applicable at $\theta = 0$, and thus no reason to set $B = 0$. In this case, a reasonable limiting initial condition is to set $\dot{\delta} = 0$ at $\theta = \pi$, meaning that the fluctuation starts growing from rest. From equation (10), this implies $B = 3\pi\delta_0/2$, and thus the evolution equation for the perturbation becomes

$$\delta_{\mathrm{MC}}(\theta) = \frac{\delta_0}{2}\left[\frac{5+\cos\theta}{1-\cos\theta} - \frac{3\theta\sin\theta}{(1-\cos\theta)^2}\right] + \frac{3\pi\delta_0}{2}\frac{\sin\theta}{(1-\cos\theta)^2}, \tag{13}$$

with the initial condition

$$\dot{\delta_{\mathrm{MC}}}\big|_{\theta=\pi} = 0. \tag{14}$$

The solution starting from this initial condition should be considered as describing the minimum possible growth rate of the density fluctuation. In reality, the buildup of the fluctuation will imply that at the initial time its growth rate is moderate but larger than zero. However, since there is no criterion to decide the initial growth rate, we consider the case of zero rate as the lower limit to the possible realistic rates, and so the evolution of actual fluctuations should be considered to be bounded from below by this case.

Equations (11) and (13), with the corresponding initial conditions given by equations (12) and (14), describe the evolution of a density fluctuation in a contracting background, in the cosmological and MC subcases, respectively. In general, we will refer to the situation of a contracting background, as an inverse Hubble flow (IHF).

Finally, we note that the collapsing background (assumed spherical) for either IHF case completes its collapse on its free-fall time, which is given by

$$\tau_{\mathrm{ff}} = \sqrt{\frac{3\pi}{32\,G\rho_0}}. \tag{15}$$

Because at $t = \tau_{\mathrm{ff}}$ the scale factor of the background a has shrunk to zero, τ_{ff} corresponds to $\theta = 2\pi$. On the other hand, we consider that the evolution starts when $\theta = \pi$, that is, at the onset of the contracting stage. Thus, relation (6) can be written as the equality

$$t = \left(\frac{\theta - \sin\theta}{\pi} - 1\right)\tau_{\mathrm{ff}}. \tag{16}$$

2.3 The Jeans case: a static background

In this next section, we will compare the evolution of the fluctuations in an IHF to the evolution of the classical Jeans case, applicable to a static background. For this case, it is well known (see, e.g. Binney & Tremaine 1987) that the general solution is

$$\delta_{\mathrm{J}}(t) = \alpha \mathrm{e}^{t/\tau} + \beta \mathrm{e}^{-t/\tau}, \tag{17}$$

where the characteristic time-scale τ is given by

$$\tau = \sqrt{\frac{1}{4\pi G\rho_0}} = \sqrt{\frac{8}{3\pi^2}}\,\tau_{\mathrm{ff}}, \tag{18}$$

and α and β are coefficients to be determined as follows. First, we note that, to compare to the cosmological IHF case, the simplest choice is $\alpha = \delta_0$ and $\beta = 0$, since we then have

$$\left(\frac{\mathrm{d}\delta_{\mathrm{J}}}{\mathrm{d}t}\right)_{t=0} = \frac{\delta_0}{\tau}. \tag{19}$$

This can be compared to the initial growth rate for the cosmological IHF case which, writing equation (12) in terms of the time variable, is

$$\left(\frac{\mathrm{d}\delta_{\mathrm{c}}}{\mathrm{d}t}\right)_{t=0} = \frac{3\pi}{8}\left(\frac{2}{3}\right)^{1/2}\frac{\delta_0}{\tau} \approx 0.962\,\frac{\delta_0}{\tau}, \tag{20}$$

and so the two initial growth rates are nearly the same.

On the other hand, in order to compare to the MC IHF solution, a useful choice is $\alpha = \beta = \delta_0/2$, since in this case the initial growth rate for the Jeans solution is zero, in agreement with the initial condition for the MC IHF solution.

3 RESULTS

Equations (11) and (13) on one hand, and equation (17) on the other, describe the linear growth of the density perturbations in an IHF and

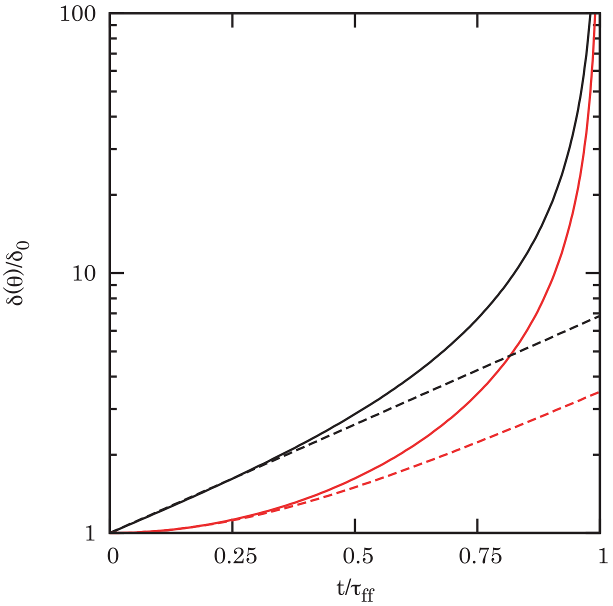

Figure 2. Linear growth of the perturbations (δ/δ_0) in an IHF (solid lines) and in a static background (the Jeans case; dashed lines) as a function of time, as given by equation (16). The cosmological case, given by equations (11) and (12) for the IHF flow, and by equation (17) with $\alpha = \delta_0$ and $\beta = 0$ for the Jeans case, is shown by the black lines. The MC case, given by equations (13) and (14) for the IHF, and by equation (17) with $\alpha = \beta = \delta_0/2$ for the Jeans case, is shown by the red lines. The time-scale in all cases is given in units of the free-fall time for the IHF background, equation (15).

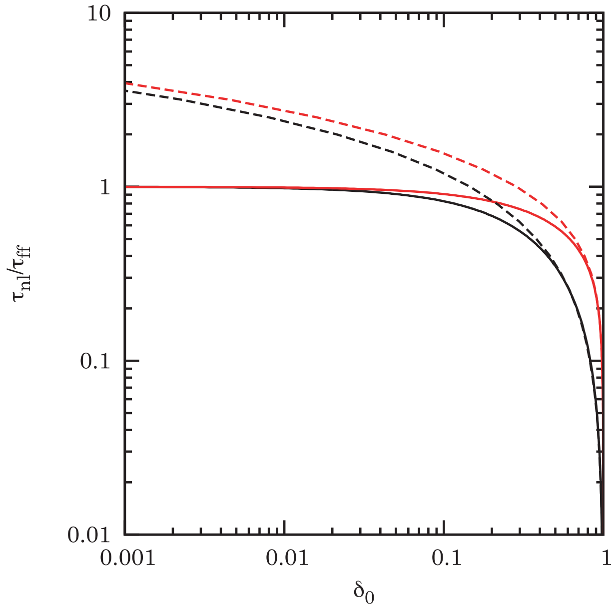

Figure 3. Time $\tau_{\rm nl}$ for the perturbation to grow to $\delta = 1$ in both the Jeans (dashed lines) and the IHF (solid lines) cases, versus the initial fluctuation amplitude, δ_0. As in Fig. 2, the black lines denote the cosmological case, and the red lines denote the MC case.

in the Jeans' case, respectively. Note that these expressions contain an explicit dependence on the initial density perturbation δ_0.

In Fig. 2, we plot the two IHF solutions, equations (11) and (13) (black and red solid lines, respectively) as a function of time, together with the Jeans solution with either finite ($\alpha = \delta_0$, $\beta = 0$) or zero ($\alpha = \beta = \delta_0/2$) initial growth rate (black and red dashed lines, respectively). The time-scale for all cases is normalized to the free-fall time, $\tau_{\rm ff}$, and all solutions are normalized to the initial fluctuation amplitude, δ_0. From Fig. 2, the linear growth of the normalized density fluctuation is seen to be faster in the IHF cases than in the Jeans case. Moreover, the fact that δ increases monotonically with time means that the density of the perturbation increases faster than that of the collapsing background.

However, note that, for the linear IHF solutions, the fluctuation terminates its collapse at the same time as the background, since, from equations (11) and (13), it is seen that $\delta \to \infty$ as $\theta \to 2\pi$, as seen in Fig. 2. The anticipated collapse of the fluctuation with respect to that of the background occurs as a consequence of the *non-linear* growth of the fluctuation, which starts when $\delta \approx 1$. After this time, the fluctuation collapses on its own free-fall time and, since its density is now roughly twice that of the background at this time, its free-fall time is now $\sim 1/\sqrt{2}$ that of the background, also at this time, implying an anticipated collapse of the fluctuation.

Fig. 3 illustrates the time for the perturbation to become non-linear ($\tau_{\rm nl}$), that is, the time for the perturbation to grow to $\delta = 1$, as a function of the initial fluctuation amplitude, δ_0. For example, consider the case of $\delta_0 = 10^{-1}$. The time for the density fluctuation to grow by a factor of $1/\delta_0$ (i.e. the time to reach non-linearity, or $\delta = 1$) in the cosmological IHF case and the corresponding Jeans case is ≈ 0.8 and $1.2\tau_{\rm ff}$, respectively, as shown by the black solid and dashed lines, respectively. For the MC case and its corresponding Jeans case with zero initial growth rate (red solid and dashed lines, respectively), these times are ~ 0.9 and $\sim 1.5\tau_{\rm ff}$, respectively. In general, this figure shows that, the smaller the initial amplitude, the larger the ratio of the Jeans to the IHF growth times. More fundamentally, the linear growth time in the Jeans case increases without limit as δ_0 decreases, while it remains finite, asymptotically approaching the free-fall time for the IHF cases. Again, this is a manifestation of the inherent non-linearity of the situation when a small-scale perturbation is growing within a larger-scale one that is also growing.

4 DISCUSSION

The results from the previous section have a number of important implications for the evolution of density fluctuations within collapsing MCs. First and foremost, our results imply that the standard notion that, in the linear regime, all fluctuations grow at the same rate, is not entirely accurate. Small-scale fluctuations embedded in a larger-scale fluctuation that is also collapsing will grow faster than isolated fluctuations. The fact that this feature has not been recognized before can be traced to the common practice of discussing purely in terms of Fourier spectra, neglecting the *phases* of the Fourier modes, which are the part related to the spatial distribution of the density field (Armi & Flament 1985). In our case, it is seen that the spatial location introduces a fundamental modification to the picture. This coupling between a small and a large scale is inevitable even in the linear regime, and arises from the fact that the large-scale fluctuation embedding the small-scale one constitutes a background which modifies the governing equations, even in the linear analysis.

In the case of structure formation within the expanding cosmological flow, it is well known that all density fluctuations are initially part of the expanding Universe, and therefore, although their corresponding density fluctuation δ is increasing monotonically, their physical size is still increasing as a consequence of the expansion, albeit at a lower rate than the global one. At some time, usually referred to as the *turnaround point*, they finally stop expanding and begin to contract, but by this time they are already non-linear with respect to the universal Hubble flow. Therefore, it is possible that the *linear* growth of a small-scale fluctuation embedded within a larger-scale one will occur while the latter is still expanding, even if its corresponding δ is increasing. In this case, the small-scale fluctuation will nevertheless find itself embedded in a background that is expanding more slowly that the global expansion, and so its growth will be faster than that of a fluctuation growing in the global expansion field, causing an earlier collapse anyway. We hope to investigate this in future work.

In fact, it is possible that the accelerated growth of density fluctuations embedded in growing density fluctuations may have already been observed in numerical simulations of halo and galaxy formation, but not recognized as a consequence of this effect. Indeed, the numerical study of the environmental dependence of halo formation by Sheth & Tormen (2004) found evidence that haloes of a given mass forming in high-density environments typically form earlier than those formed in low-density environments. This result has been subsequently confirmed by other authors (e.g. Harker et al. 2006; Wechsler et al. 2006; Maulbetsch et al. 2007; Wang et al. 2011), some of which use a different measure of the environment's overdensity.[1] We suggest that this result may be a consequence of the halo being located in an overdense region which is itself growing; therefore, it grows faster than otherwise.

In the MC context, our results have also relevant implications. First, if the formation mechanism of MCs is such that they quickly acquire many Jeans masses, then their collapse is nearly pressureless (as suggested by many numerical studies; see, e.g. Vázquez-Semadeni et al. 2007, 2011; Heitsch & Hartmann 2008; Heitsch et al. 2008; Banerjee et al. 2009; Heitsch, Ballesteros-Paredes & Hartmann 2009; Micic et al. 2013), and then even moderately linear perturbations, with $\delta_0 \gtrsim 10^{-1}$ can grow faster than the global collapse, as systematically observed in those simulations. Of course, this is supplemented by the facts that (a) the turbulence may produce moderately *non-linear* fluctuations, which then have shorter free-fall times even from the start, and (b) that flattened or filamentary structures have longer free-fall times than spherical ones, so that spheroidal fluctuations within such structures will collapse earlier than the larger structure (Pon et al. 2012; Toalá et al. 2012). But the important issue here is that, even in the worst-case scenario for fragmentation, namely that of a spherical geometry with linear density fluctuations, the perturbations are able to collapse earlier than the cloud.

5 SUMMARY AND CONCLUSIONS

Recent numerical simulations of MC formation and evolution have shown that the clouds engage in global gravitational collapse, and that the density fluctuations within them grow and complete their local collapse before the global collapse is completed (Vázquez-Semadeni et al. 2007, 2011; Heitsch & Hartmann 2008; Heitsch et al. 2008, 2009; Banerjee et al. 2009; Micic et al. 2013). Motivated by these results, in this paper we have performed a linear analysis of the growth rate of density fluctuations immersed in a medium that is itself undergoing global gravitational contraction. We have used the standard linear analysis used for the growth of density fluctuations in Hubble flows, but considering the case of an IHF, where the background is contracting rather than expanding. We considered two variants of an IHF. The first is appropriate for a cosmological setup, in which the initial fluctuation at the onset of the collapse of the background is already growing at a finite rate, consistent with the evolution during a previous epoch of growth, during the epoch when the background was still expanding. The other is meaningful as a lower limit for a collapsing MC, in which we assume the initial growth rate to be zero. This represents a lower limit to the possible initial growth rate of the fluctuation, which is unconstrained in this case.

Our main results are the following.

(i) Density fluctuations embedded in an IHF grow faster than in the standard Jeans analysis (where the background is static).

(ii) While in the Jeans case the growth time to reach non-linearity (i.e. to reach a density fluctuation amplitude $\delta = 1$) increases without limit as $\delta_0 \to 0$, where δ_0 is the initial value of δ, in the IHF case this growth time is bounded from above by the free-fall time of the background density, $\tau_{\rm ff}$. This reflects the fact that the density fluctuation is embedded in a medium which is itself collapsing on a time-scale $\tau_{\rm ff}$, and the longest possible time for the fluctuation to complete its collapse is the free-fall time of the medium in which it is embedded.

(iii) For perturbations embedded in an IHF having initial amplitudes $\delta_0 \gtrsim 10^{-1}$, the time to reach non-linearity is $\sim 0.8\tau_{\rm ff}$ in the cosmological variant, and $\sim 0.9\tau_{\rm ff}$ in the MC variant. From then on, the perturbation grows at its own free-fall rate, and therefore it will always be 'ahead' of the global collapse.

In the context of large-scale structure formation in the Universe, these results may offer an alternative (or complementary) explanation for the observation in numerical simulations that haloes of a given mass typically form earlier in high-density environments than those formed in low-density environments. In the MC context, they may offer an explanation for the ubiquitous observation in numerical simulations that small-scale density fluctuations within large MCs complete their collapse (i.e. form stars) before the collapse of the parent cloud is completed, even if the time difference is not too large. It must also be borne in mind that the spherical symmetry that we have assumed for the background constitutes a worst-case scenario for the possibility of fragmentation, since in this case the free-fall time depends only on the density, and is independent of size scale. Instead, for flattened or filamentary geometries, the actual collapse time is larger than the spherical free-fall time by factors that depend on the aspect ratio of the object (Pon et al. 2012; Toalá et al. 2012).

More fundamentally, our results show that the hierarchical nesting of the fluctuations of different scales affects the growth rate of density fluctuations, even in the linear regime. Thus, our results imply that density perturbations are always able to collapse earlier than the whole cloud, as envisioned by Hoyle (1953), and recently observed in the numerical simulations, regardless of whether they have linear or non-linear amplitudes, making Hoyle-like fragmentation an inescapable process in multi-Jeans-mass MCs.

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[1] Note, however, that this effect is weak compared to the dependence on the mass assembly history.

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APPENDIX A: FRIEDMANN'S EQUATION FOR A SPHERICAL MC

Let us suppose a spherical cloud with total mass M and initial radius $R(t = 0) = R_0$. By energy conservation we have

$$\frac{1}{2}v(t)^2 - \frac{GM}{R(t)} = E_{\rm TOT}, \tag{A1}$$

and $E_{\rm TOT}$ can be evaluated at $t = 0$, supposing that $v(t) = 0$. This gives

$$E_{\rm TOT} = -\frac{GM}{R_0}. \tag{A2}$$

Manipulating equation (A1) we can write

$$\frac{v(t)^2}{R(t)^2} - \frac{2GM}{R(t)^3} = -\frac{2GM}{R_0}\frac{1}{R(t)^2}, \tag{A3}$$

and using $M/R(t)^3 = 4/3\pi\rho(t)$, we can rewrite the former equation as

$$\frac{v(t)^2}{R(t)^2} - \frac{8\pi G\rho(t)}{3} = -\frac{8\pi G\rho_0}{3}\frac{R_0^2}{R(t)^2}. \tag{A4}$$

If we now define $a(t) = R(t)/R_0$, this is, $\dot{a(t)} = \dot{R}(t)/R_0$ with $v(t) = \dot{R}(t)$, we can write equation (A4) as

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{8\pi G\rho_0}{3}\frac{1}{a^2} = \frac{8\pi G\rho(t)}{3}. \tag{A5}$$

Under this scenario, we can define $k \equiv 8/3\pi G\rho_0$ and write

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G\rho(t)}{3}. \tag{A6}$$

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