Statistics for Astronomers
Homework #3 (Due before 5:00 PM on Wednesday, 2020.10.21)

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October 14, 2020

Notes: (1) You are welcome to use `Python` functions to evaluate probabilities for various distributions, and `Mathematica/Wolfram Alpha` to compute integrals if necessary. **Just mention your source in each case!** (2) Email me your `Python` scripts and any/all resulting output plots/images.

1. Assume that the Initial Mass Function of stars is of the form

$$p_M(m) = C\ m^{-\alpha} \qquad \alpha > 1,\ m_1 \leq m \leq m_2$$

   (a) **(2 points)** Compute C.

   (b) **(5 points)** A data set consists of independently-determined masses $m_i (i = 1, 2, \cdots, N)$ for N stars. What are the expressions for the maximum likelihood estimates for m_1 and m_2 in terms of the data?

   (c) **(4 points)** If the sample mean of $\ln m$ (m in $\mathrm{M}_\odot$) for the data is 0.8, and $\widehat{m_1} = 1\ \mathrm{M}_\odot$, $\widehat{m_2} = 200\ \mathrm{M}_\odot$, write down an equation for $\widehat{\alpha}$ and find a numerical solution with a Python script.

2. Assume that the probability p per unit time of a supernova event is independent of the properties of the host galaxy. In a dataset of N galaxies, the i^{th} galaxy was observed for time t_i resulting in n_i SNe observations.

   (a) **(2 points)** What is the maximum likelihood estimate for p?

   (b) **(2 points)** What is the Cramér-Rao bound on the variance of this estimate?