Stellar Atmospheres: Lecture 11, 2020.06.01

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References

1. Lamers & Casinelli, *Introduction to Stellar Winds*, Ch. 1-3

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Observational evidence for winds:
Sun – cometary tails, magnetosphere (van Allen Belts), aurorae $\cdots$
Hot stars – P Cygni profiles of highly-ionised species. P Cygni is a luminous blue variable (LBV) $\sim 6 \times 10^5$ L$_\odot$.
Cool stars – expansion velocity $\to$ Doppler-shifted lines $\to$ asymmetric line profiles.
  Infrared excess due to reprocessing of radiation by circumstellar dust.
  Parabolic or double-horned line profiles in the sub-mm/radio.

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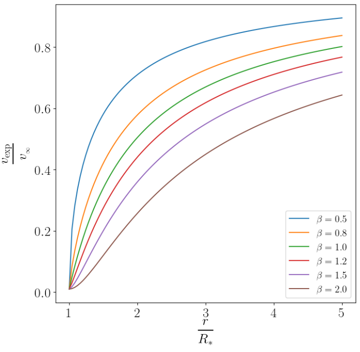

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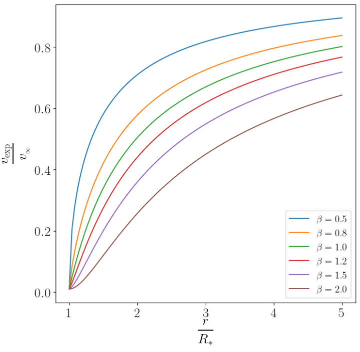

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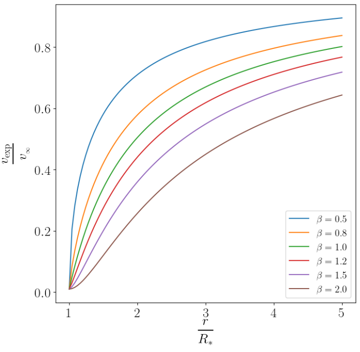

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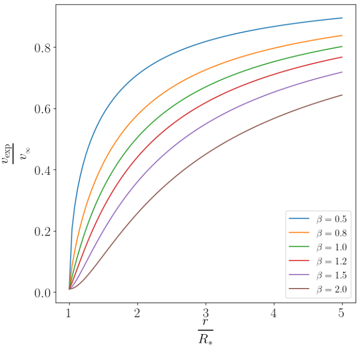

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Ψ = gas:dust mass ratio $\sim 10^2 - 10^3$, but grains drag molecules out with them via momentum coupling.

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3. Wave-driven wind

   Shocks (*e.g.*, due to pulsations), acoustic, MHD/magnetoacoustic waves. Inefficient on their own, but can heat up material to increase pressure or increase density of material to enhance interaction with radiation.

4. Rotation

   Transfer of angular momentum in rotating systems – outflowing disks from rotating stars, magnetic rotators.

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   Masers seen in many astrophysical environments. Stellar outflows: massive AGB stars (OH/IR stars).

Stationary wind

Continuity Equation for the material in the outflow:

$$\frac{d\rho(r)}{dt} = \frac{\partial \rho(r)}{\partial t} + \nabla \cdot (\overbrace{v(r)\rho(r)}^{\text{mass flux}}) = 0$$

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The velocity dependence is usually such that the material is accelerated beyond the sonic speed and escape velocity within a small zone close to the star, and a terminal velocity v_∞ is achieved. In this region, a stationary wind implies an inverse-square density profile.

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Energy equation: Simplest model is isothermal, $T =$ constant.

Isothermal pressure-driven stationary wind - I

Temperature constant in the wind layer, pressure gradient and gravity are only external forces on the gas.

One of the simplest models. Easily solved. Can study how v_∞ and ρ depend on the forces.

MLR for stationary wind model uniquely determined by boundary conditions at $r = R_*$.

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Example: $T_{\rm corona} \approx 10^6$ K, $v_{\rm esc}(R_\odot) \approx 600$ km s^{-1}, $r_c = 3.5\ R_\odot$

If no coronal heating: $T \approx 6000$ K, and $r_c > 1000\ R_\odot$.

Solar wind would still exist, but accelerated much slower (sonic point at ≈ 6 AU).

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For an isothermal outflow, $r_c \geq r_0 \implies v_{\rm esc}(r_c) \geq 2c_s$ (sonic point $<$ escape point) and $\dfrac{d \ln v}{dr} \geq 0$.

The critical solution is the only solution satisfying these criteria.

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$$\frac{d \ln v}{dr} = \frac{1}{v^2 - c_s^2}\left[\frac{2c_s^2}{r} - \frac{GM}{r^2}\right].$$

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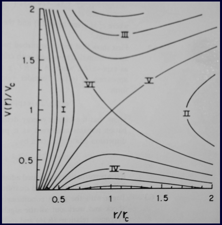

Rai Chouduri, *The Physics of Fluids and Plasmas*

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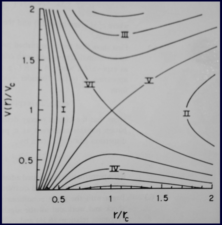

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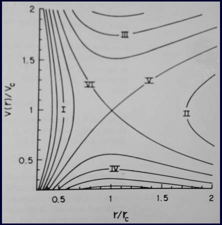

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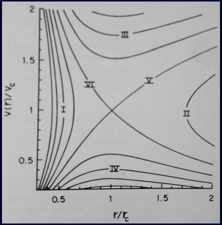

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Near surface ($\xi < 1$), $\eta \approx \xi^{-2} \exp\left[-2/\xi + 3/2\right]$.
At large distances ($\xi \gg 1$): $\eta \sim \sqrt{\ln \xi}$. The outflow velocity diverges, which is unphysical. True behaviour deviates from the isothermal model prediction (T drops, v becomes constant, ρ falloff steeper than hydrostatic solution beyond critical point).

Limitation of isothermal pressure-driven winds

Consider an O star: $T_{\rm eff} \approx 40\,000$ K, $M \approx 40$ $M_\odot$, $R \approx 20 R_\odot$ (Carroll & Ostlie, Appendix G).

Compute: $c_s \approx 20$ km s^{-1}, $v_{\rm esc}(R_*) \approx 600$ km s^{-1} $\Longrightarrow r_c \approx 220 R_*$.

Observed sonic point: very close to R_*.

$\Longrightarrow$ additional force required to oppose strong gravitational pull (radiation pressure, magnetic fields).

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Non-isothermal wind: temperature gradient will change c_s and hence location of critical point, and therefore MLR.

Maintaining a temperature gradient requires higher pressure gradients which have to be sourced from other mechanisms.

Radiation-driven winds

Cool stars: continuum absorption by freshly-formed dust. Hot stars: resonance scattering from ionised metal lines.

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Momentum balance: momentum of the wind is obtained from photon absorption.

$$\dot{M}v_\infty = \overbrace{\frac{L_*}{c}}^{\text{momentum/time}} \overbrace{(1 - e^{-\tau})}^{\text{abs. fraction}} \Longrightarrow \frac{\dot{M}}{10^{-7}\mathrm{M}_\odot\ \mathrm{yr}^{-1}} \approx 200 \left(\frac{L_*}{10^4\mathrm{L}_\odot}\right) \left(\frac{v_\infty}{10\ \mathrm{km\ s}^{-1}}\right)^{-1} (1 - e^{-\tau}).$$

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$$\frac{dp_{\mathrm{rad}}}{dz} = \frac{4\pi}{c} \int \alpha_\nu^{\mathrm{ext}} \frac{dK_\nu}{d\tau_\nu} d\nu = \frac{1}{c} \int \alpha_\nu^{\mathrm{ext}} \frac{d}{d\tau_\nu} (4\pi K_\nu)\, d\nu = \frac{1}{c} \int \alpha_\nu^{\mathrm{ext}} F_\nu \, d\nu.$$

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$$\frac{dp_{\mathrm{rad}}}{dz} = \frac{4\pi}{c} \int \alpha_\nu^{\mathrm{ext}} \frac{dK_\nu}{d\tau_\nu} d\nu = \frac{1}{c} \int \alpha_\nu^{\mathrm{ext}} \frac{d}{d\tau_\nu} (4\pi K_\nu) \, d\nu = \frac{1}{c} \int \alpha_\nu^{\mathrm{ext}} F_\nu d\nu.$$

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Radiation-driven winds

Cool stars: continuum absorption by freshly-formed dust. Hot stars: resonance scattering from ionised metal lines.

Radiation pressure and the Eddington Limit

Momentum balance: momentum of the wind is obtained from photon absorption.

$$\dot{M} v_\infty = \overbrace{\frac{L_*}{c}}^{\text{momentum/time}} \overbrace{(1 - e^{-\tau})}^{\text{abs. fraction}} \Longrightarrow \frac{\dot{M}}{10^{-7}\mathrm{M}_\odot\ \mathrm{yr}^{-1}} \approx 200 \left(\frac{L_*}{10^4 \mathrm{L}_\odot}\right) \left(\frac{v_\infty}{10\ \mathrm{km\ s}^{-1}}\right)^{-1} (1 - e^{-\tau}).$$

Example: $L = 5000\ \mathrm{L}_\odot$, $v_\infty = 15\ \mathrm{km\ s}^{-1}$, $\tau = 0.01 \Longrightarrow \dot{M} = 6.7 \times 10^{-8}\ \mathrm{M}_\odot\ \mathrm{yr}^{-1}$.

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Star blows away surface material if $L > L_{\rm Edd} \equiv \dfrac{4\pi GMc}{\overline{\kappa}} = 3.2 \times 10^4 \left(\dfrac{M}{\mathrm{M}_\odot}\right)\left(\dfrac{\overline{\kappa}}{0.039\ \mathrm{m}^2\ \mathrm{kg}^{-1}}\right)^{-1} \mathrm{L}_\odot$,

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AGB and RSG stars have $L \sim 0.1 - 0.3 L_{\mathrm{Edd}}$. η Car is an example with $L \sim L_{\mathrm{Edd}}$.

Dust-driven winds in cool stars

$v_\infty \sim 10 - 30$ km s^{-1}, $T_{\rm eff} \sim 2700 - 4000$ K. AGB ($1 - 5$ M$_\odot$, $3000 - 10^5$ L$_\odot$) and RSG ($5 - 25$ M$_\odot$, $10^5 - 10^6$ L$_\odot$).

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Pulsators. Semi-regular variables (small amplitude, $P < 100$ d), long-period variables (large amplitude, fundamental mode, $P \sim 100 - 300$ d). Mira-type stars are LPVs. Pulsations can also drive a weak wind.

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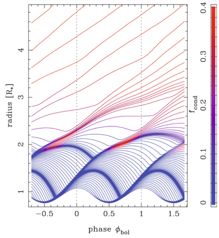

Nowotny et al. 2010 A&A 514, A35

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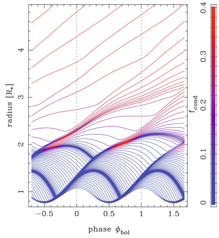

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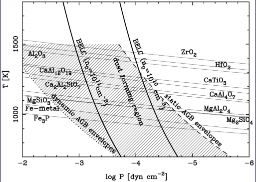

Elvis et al. 2002 ApJ 567, L107

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AGB star atmospheres start out oxygen-rich. Stars with masses $2 - 4$ M$_\odot$ become carbon-rich by the third dredge-up process. The chemistry of the molecules and dust in the envelope is regulated accordingly.

O-rich dust: refractory oxides (Al, Mg), amorphous silicates (olivine, pyroxene), crystalline silicates (enstatite).

C-rich dust: amorphous carbon, diamond/graphite, silicon carbide, MgS, hydrogenated amorphous carbons (HACs).

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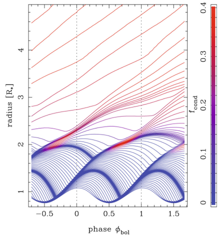

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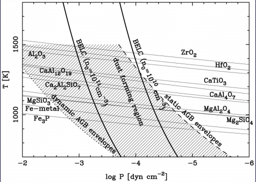

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The MLR depends on the chemistry – in general, carbonaceous dust has higher κ and hence is more efficient in absorbing radiation. Silicate dust requires iron to enhance its opacity (*e.g.*, Höfner 2007 ASPC 378, 145).